

§15 $(x, y) \rightarrow \max$ и $\min$: $y'(x) = f(x, y)$; $f(x, y) = 0$
 $f'_x(x, y) > 0 (< 0)$

§16a

$$y' = y - x^2, \quad y'' = (y - x^2)'_x = y' - 2x = 0, \quad y' = 2x = y - x^2,$$

$$y = 2x + x^2.$$

§17

$$y = e^{cx}, \quad y' = e^{cx} \cdot c = y \cdot c, \quad c = \frac{\ln y}{x} \Rightarrow y' = \frac{\ln y}{x} \cdot y.$$

§21

$$x^2 + cy^2 = 2y, \quad 2x + 2cyy' = 2y', \quad y' = \frac{x}{1-2c}, \quad c = \frac{2y-x^2}{y^2} \Rightarrow$$

$$\Rightarrow y' = xy.$$

§30

Составить групп. ур-ие вы. раз. 1 участн коэф. делит на предел $y = 2x$;

$$(x-x_0)^2 + (y-2x_0)^2 = 1,$$

$$2(x-x_0) + 2(y-2x_0)y' = 0 \Rightarrow x_0 = \frac{x+yy'}{1+2y'}; \quad \left(x - \frac{x+yy'}{1+2y'}\right)^2 + \left(y - \frac{2(x+yy')}{1+2y'}\right)^2 = 1$$

§52

$$\sqrt{y^2+1} dx = xy dy, \quad \frac{dx}{x} = \frac{y dy}{\sqrt{y^2+1}} = \int \frac{dx}{x} \quad \left(\sqrt{y^2+1} = \ln|x| + C \right), \quad x=0$$

сменить переменную.

§54

$$y' \operatorname{ctg} x + y = 2, \quad y(x) \rightarrow -1; x \rightarrow 0.$$

$$\operatorname{ctg} x = \frac{2-y}{y'}; \quad \frac{dy}{dx} \operatorname{ctg} x + y = 2, \quad \frac{\operatorname{ctg} x}{dx} = \frac{2-y}{dy}; \quad \frac{dx}{\operatorname{ctg} x} = \frac{y dy}{y-2}$$

$$\int \frac{dy}{y-2} = - \int \frac{\sin x dx}{\cos x}; \quad \ln|y-2| = \int \frac{d(\cos x)}{\cos x}, \quad \ln|y-2| = \ln|\cos x| + \ln C$$

$$y = \cos x + 2. \quad C = -3$$

$$y = 2 - 3 \cos x.$$

§56

$$xy' + y = y^2, \quad y(1) = 0.5$$

$$x \frac{dy}{dx} + y = y^2, \quad \frac{dy}{y^2-y} = \frac{dx}{x}; \quad \int \frac{dy}{y-1} - \int \frac{dy}{y} = \int \frac{dx}{x},$$

$$\ln|y-1| - \ln|y| = \ln|x| + C, \quad \ln\left|\frac{y-1}{y}\right| = \ln|x|, \quad \frac{y-1}{y} = cx$$

$$\begin{cases} y-1 = cxy \\ y=0 \end{cases}, \quad C = -1, \quad \text{при } y(1) = 0.5; \quad y(1+x) = 1.$$

560

$$z' = 10^{x+z}; \quad \frac{dz}{dx} = 10^x \cdot 10^z; \quad \int \frac{dz}{10^z} = \int \frac{dx}{10^x} = \frac{10^x}{\ln 10} + C$$

$$\frac{10^{-z}}{\ln 10}; \quad -10^{-z} = 10^x + C; \quad 10^x + 10^{-z} = C$$

564

$$(x+2y)y' = 1; \quad u(x) = x+2y, \quad y' = \frac{u'-1}{2}; \quad \frac{u(u'-1)}{2} = 1; \quad u(\frac{du}{dx} - 1) = 2$$

$$u \frac{du}{dx} - u = 2, \quad \frac{u du}{u+2} = dx, \quad \int \frac{u du}{u+2} = \int dx; \quad \int \frac{u-2}{u+2} = x+C$$

$$\Rightarrow \begin{cases} 2y - \ln|x+2y+2| = C \\ x+2y = -2 \end{cases} \quad \text{Ans: } x+2y = -2$$

5102

$$(x-y)dx + (x+y)dy = 0, \quad y = tx, \quad \text{even } x\text{-comp. no separation}$$

$$(x-tx)dx + (x+tx)(t dx + x dt) = 0; \quad (1-t)dx + (1+t)t dx + (1+t)x dt = 0$$

$$(1+t)x dt = -(1+t^2)dx, \quad \frac{(1+t)dt}{1+t^2} = -\frac{dx}{x}, \quad \text{Ans: } \arctan t + \frac{1}{2} \ln(1+t^2) = -\ln|x| + C$$

$$\arctan \frac{y}{x} + \frac{1}{2} \ln(x^2+y^2) = C$$

5106

$$(x^2+y^2)y' = 2xy, \quad y = tx; \quad (x^2+t^2x^2)(t'x+t) = 2xt^2$$

$$\frac{dt}{\frac{t^2-1}{t^2+t}} = \frac{dx}{x}; \quad \int \frac{dt(1+t^2)}{t^3-t} = -\int \frac{dx}{x}; \quad \frac{1+t^2}{t^3-t} = \frac{A}{t} + \frac{B}{t-1} + \frac{C}{t+1}$$

$$-\ln|t| + \ln|t-1| + \ln|t+1| = -\ln|x| + \ln C$$

$$\frac{t^2-1}{t} = \frac{C}{x} \cdot \frac{(t^2-1)x}{y} = \frac{C}{x}, \quad \int \frac{y^2 - cy}{y} = x^2; \quad y=0, y=x, y=-x$$

515

515, b)

$$5) \quad y' = x - e^y; \quad y'' = 0, \quad y'' = 1 - e^y \cdot y' = 0 = 1 - e^y(x - e^y) = 0$$

$$x = e^y$$

$$x = \frac{1+e^{2y}}{e^y}$$

$$b) x^2 + y^2 y' = 1, \quad 2x + 2y(y')^2 + y' y'' = 0, \quad y'' = -\frac{2x + 2y(y')^2}{y'} = 0$$

$$y = \frac{1-x^2}{y'}; \Rightarrow 2x + 2y \cdot \frac{(1-x^2)^2}{y'^3} = 0, \quad \Rightarrow xy^3 = -(1-x^2)^2$$

530

$$y = \sin(x+C), \quad y' = \cos(x+C);$$

$$\arcsin y = x+C, \quad C = \arcsin y - x \Rightarrow y' = \cos(\arcsin y) = \pm \sqrt{1-x^2}$$

532

$$y^2 + Cx = x^3, \quad C = \frac{x^3 - y^2}{x}; \quad 3y^2 y' + C = 3x^2, \quad y' = \frac{3x^2 - C}{3y^2} = \frac{3x^2 - \frac{x^3 - y^2}{x}}{3y^2} =$$

$$= \frac{\frac{2x^3 + y^2}{x}}{3y^2} = \frac{2x^3 + y^2}{3xy^2} = \frac{2x^2 + y^2}{3xy}$$

533

$$ax^2 + bx + c = y, \quad ax^2 + bx = y, \quad y' = 2ax + b, \quad y'' = 2a, \quad a = \frac{1}{2} y''$$

$$b = y' - 2ax = y' - y''x$$

$$\frac{1}{2} y'' x^2 + (y' - y''x)x = y, \quad y''x^2 - 2xy' + 2y = 0$$

54

$$xy dx + (x+1)dy = 0, \quad x(y dx + dy) = -(x+1)dy, \quad -\frac{x dx}{x+1} = \frac{dy}{y}$$

$$x = -1 - \ln u; \quad \int \frac{dx}{y} = -\int \frac{x}{x+1} dx, \quad \ln|y| = -\int (1 - \frac{1}{x+1}) dx = -x + \ln|x+1| + C$$

$$|y| = e^{-x + \ln|x+1| + C}; \quad y = e^{-x} (x+1) e^C, \quad x = -1$$

55

$$(x^2-1)y' + 2xy^2 = 0, \quad y(0) = 1. \quad (x^2-1) \frac{dy}{dx} = -2xy^2, \quad \int \frac{-2x}{x^2-1} dx = \int \frac{dy}{y^2}$$

$$-\int \frac{2x}{x^2-1} dx; \quad \frac{2x}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1} \Rightarrow A=1, B=1 \Rightarrow +(\ln|x+1| + \ln|x-1|) = +\frac{1}{y} + C$$

$$C = -1; \Rightarrow y = \frac{1}{\ln|x+1| + \ln|x-1| + 1}$$

55

$$y' = 3\sqrt[3]{y^2}; \quad y(2) = 0; \quad \frac{dy}{dx} = 3\sqrt[3]{y^2}; \quad \int \frac{dy}{3\sqrt[3]{y^2}} = \int 3 dx; \quad y \cdot y^{\frac{1}{3}} = 3x + C; \quad y^{\frac{4}{3}} = 3x + C; \quad C = -2; \quad y^{\frac{4}{3}} = 3x - 2; \quad y^3 = (x-2)^3$$

63

$$y' - y = 2x - 3; \quad \frac{dy}{dx} - y = 2x - 3, \quad dy - (dx)y = (2x-3)dx;$$

$$dy = (2x-3+y)dx; \quad z = y + 2x - 3; \quad dz = dy + 2dx$$

$$\frac{dz}{dx} = z; \quad \ln|z+2| = x + \ln C, \quad y = 1 - 2x + Ce^x$$

55

$$y' = \sqrt{4x+2y-1}; \quad \left(\frac{dy}{dx}\right)^2 = 4x+2y-1; \quad (dy)^2 = (4x+2y-1)(dx)^2$$

$$dz = 4dx + 2dy; \quad dy = \frac{1}{2}(dz - 4dx)$$

$$\frac{1}{4}(dz - 4dx)^2 = 2(dx)^2; \quad (dz)^2 - 8dzdx + 16(dx)^2 = 4z(dx)^2 \quad | : (dx)^2$$

$$t = \frac{dz}{dx} \Rightarrow t^2 - 8t + 16 - 4z = 0; \quad D = 64 - 64 + 16z \quad t_{1,2} = \frac{8 \pm \sqrt{16z}}{2} =$$

$$\frac{dz}{dx} = 4 \pm 2\sqrt{z}; \quad \int \frac{dz}{4 \pm 2\sqrt{z}} = \int dx \Rightarrow \dots = 4 \pm 2\sqrt{z}$$

$$\int \frac{dz}{4 \pm 2\sqrt{z}} = \left| \ln \frac{z+u}{z-u} \right| = \frac{1}{2} \int \frac{2udu}{u^2} = \frac{1}{2} \int \frac{(u^2-2)-2}{u^2} du = \int \frac{du}{u} - 2 \int \frac{du}{u^2} = \ln|u| + \frac{2}{u} =$$

$$\ln|z+2\sqrt{z}| + 2 = x+C; \quad \sqrt{4x+2y-1} + 2 \ln|\sqrt{4x+2y-1} + 2| = x+C.$$

105

$$y' + x^2 y' = xy y'; \quad y = tx, \quad d(x^2 + x^2 t dx + x dt) = x(tx)(t dx + x dt)$$

$$t dx + x dt = t^2 x dx + t dx + x dt = 0$$

$$t^2 x^2 + x^2(t'x + t) = tx^2(t'x + t), \quad x^2(t'x + t) = tx^3 t',$$

$$t'x + t = tx t', \quad x(t-1)t' = t, \quad \frac{dt}{dx} x(t-1) = t, \quad \frac{t-1}{t} dt = \frac{dx}{x}$$

$$\int \frac{t-1}{t} dt = \int \frac{t}{t} dt - \int \frac{1}{t} dt = t - \ln t; \quad t - \ln t = \ln Cx, \quad \ln Cxt = t,$$

$$\ln Cy = \frac{t}{x}, \quad y = Ce^{\frac{y}{x}}.$$

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$$xy' - y = x \lg \frac{y}{x}, \quad y = tx; \quad x(t'x + t) - tx = x \lg \frac{tx}{x};$$

$$t'x = \lg t, \quad x \frac{dt}{dx} = \lg t, \quad \int \frac{dt}{\lg t} = \int \frac{dx}{x} = \int \frac{\cos t}{\sin t} dt = \int \frac{d \sin t}{\sin t} = \ln|\sin t| + C$$

$$\ln|x| = \ln|\sin t| + C, \quad \ln|x| = \ln|\sin \frac{y}{x}| + C, \quad Cx = \sin \frac{y}{x}.$$

111

$$(y + \sqrt{xy})dx = xdy, \quad y = tx; \quad tx + \sqrt{tx^2} = x(t'x + x); \quad t \pm \sqrt{t} = t'x + x$$

$$\pm \frac{dt}{t} = \frac{dx}{x}; \quad \pm 2t^{0.5} = \ln Cx, \quad \pm 2\sqrt{xy} = \ln Cx;$$

$$\Rightarrow 2\sqrt{xy} = \ln Cx, \quad x > 0, y > 0.$$

112

$$xy' = \sqrt{x^2 - y^2} + y; \quad y = tx; \quad x(t'x + t) = \sqrt{x^2 - t^2 x^2} + tx \quad x > 0 \text{ пар.}$$

$$t'x + t = \pm \sqrt{1 - t^2} + t$$

$$\int \frac{dt}{\sqrt{1-t^2}} = \int \frac{dx}{x}$$

$$\pm \arcsinh t + c = \ln |x|; \quad t = \pm 1 \text{ perm.? } \Rightarrow y = \pm x - \text{perm.}$$

$$\pm \arcsinh \frac{y}{x} = \ln c x$$

Komp.

S113

$$(2x-4y+6)dx + (x+y-3)dy = 0$$

$$\begin{cases} 2x-4y+6=0 \\ x+y-3=0 \end{cases} \Rightarrow \begin{cases} y=2 \\ x=1 \end{cases} \quad \begin{matrix} x=1+u \\ y=2+v \end{matrix} \quad \begin{matrix} dx=du \\ dy=dv \end{matrix}$$

$$(2u-4v)du + (u+v)dv = 0, \quad (2u-4v)du + (u+tv)(u dt + t du) = 0$$

$$v=tu, \quad dv = t du + u dt \quad 2du - 4tdv + u dt + t du + t u dt + t^2 du = 0$$

$$(2-3t+t^2)du = -(1+t)u dt, \quad -\frac{du}{u} = \frac{(1+t)t}{t^2-3t+2}$$

$$\frac{1+t}{t^2-3t+2} = \frac{t+1}{(t-2)(t-1)} = \frac{A}{t-2} + \frac{B}{t-1}$$

$$\begin{matrix} A=3 \\ B=-2 \end{matrix}$$

$$-\ln|u| = 3\ln|t-2| - 2\ln|t-1| + c$$

$$\ln\left(\frac{1}{|u|}\right) = \ln\left(\frac{(t-2)^3}{(t-1)^2} c\right)$$

$$c \frac{1}{u} = \frac{c(t-2)^3}{(t-1)^2}, \quad t = \frac{y}{x}$$

$$\left(\frac{y}{x}-1\right)^2 = c u \left(\frac{y}{x}-2\right)^3, \quad (v-u)^2 = c(v-2u)^3$$

$$(y-x+1)^2 = c(y-x-2x+2)^3, \quad y=2x$$

Ansatz: $t=1$,
 $u=0$

S114

$$(2x+y+1)dx - (4x+2y-3)dy = 0$$

$\int u = \infty$ - perm.

$$u = 2x+y+1, \quad y = u-2x-1, \quad dy = du - 2dx$$

$$u dx - (2u-5)(du-2dx) = 0$$

$$-(2u-5)du = dx(4u-10-u) = 0$$

$$-(5-2u)du + (4u-10)dx = 0$$

$$(5u-10)dx = (2u-5)du$$

$$dx = \frac{(2u-5)du}{(5u-10)}$$

$$x+c = \frac{1}{5} \int \frac{2u-5}{u-2} du$$

$$x+c = \frac{1}{5} \left(\int \left(\frac{2u-4}{u-2} - \frac{1}{u-2} \right) du \right)$$

$$x+c = \frac{1}{5} (2u - \ln|u-2|),$$

$$\ln|u-2| = 2u - 5x + c, \quad u-2 = e^{2u-5x}$$

$$2x+y+1 = c e^{4x+2y+2-5x} + 2, \quad \frac{2x+y-1}{2} = c e^{2y-x+2}$$

Ansatz: $u=2$ perm.

S125

$$2y' + x = 4y, \quad y = 2^m$$

$$\begin{matrix} m-1 & 1 & m/2 \end{matrix}$$

$$m-1 = 1 = \frac{m}{2}$$

$$\Rightarrow m=2 \Rightarrow y = 2^2$$

Revenance:

$$4z z' + x = 4z,$$

$$4z \frac{dz}{dx} + x = 4, \quad \frac{dz}{dx} = \frac{4z-x}{4z}, \quad z = tx,$$

$$dz = t dx + 1 dx,$$

$$\frac{x dt + t dx}{dx} = \frac{4t-1}{4t},$$

$$4t^2 dx + 4t^2 dx = x^2 dt - dx,$$

$$4dt \cdot tx = dx(-4t^2 + 4t - 1)$$

$$-\frac{4dt}{4t^2 - 4t + 1} = \frac{dx}{4x} - \frac{\int 4t dt}{x(2t-1)^2} =$$

$$= \int \frac{dx}{x}; \ln|x| + c = -\ln|t| + \frac{1}{4}$$

$$\ln|t| = c + \frac{1}{4}, \quad tx = Ce^{\frac{1}{4}};$$

$$u = 2t - 1$$

$$t = \frac{z}{x}$$

$$(2t-1)x = Ce^{\frac{2t-1}{4}}$$

$$(2\frac{z}{x}-1)x = Ce^{\frac{2z-x}{4}}$$

$$2z-x = Ce^{\frac{2z-x}{4}}, \quad (2y-x) = Ce^{\frac{x}{2y-x}}$$

Реш. $t = \frac{1}{2}, x = 0;$
 Если $c = 0$ не бер. В противном случае $c \neq 0$

линейное уравнение 1-го порядка

$$y'(x) + a(x)y(x) = f(x) \quad (\text{линейное уравнение})$$

$$1) y'(x) + a(x)y(x) = 0$$

$$\frac{dy}{y} = -a(x)dx, \quad y(x, c)$$

§136

$$xy' - 2y = 2x^4, \quad xy' - 2y = 0$$

$$x(c'x^2 + 2cx) = 2x^4$$

$$-2cx^2 = 2x^4$$

$$c'x^3 + 2cx^2 - 2x^4 = 2x^4$$

$$\frac{dy}{y} = \frac{2dx}{x}$$

$$\ln|y| = 2\ln|x| + c$$

$$y = cx^2$$

$$y(x) = c(x)x^2$$

$$\int c' dx, \quad c = x^2 + \tilde{c}, \quad y = x^4 + \tilde{c}x^2$$

§139

$$(xy + e^x)dx - xdy = 0$$

$$(xy + e^x) - xy' = 0,$$

$$x(y' - y) = e^x, \quad y' = \frac{e^x}{x} + y$$

$$\frac{dy}{y} = dx + \frac{e^x}{x} dx$$

$$\ln|y| = x + c, \quad y = e^x C(x), \quad y' = e^x + C(x)e^x$$

$$xe^x(C + C') - xe^x C = e^x$$

$$xC + xC' - xC = 1, \quad x \frac{dC}{dx} = 1,$$

$$dC = \frac{dx}{x} \Rightarrow C = \ln|x| + \tilde{C},$$

$$y = e^x(\ln|x| + \tilde{C}), \quad x = 0 \quad (\text{генератор})$$

§145

$$(x + y^2)dy = ydx, \quad 1 \cdot dy$$

$$(x + y^2) = yx', \quad x' - \frac{1}{y}x = -y;$$

$$\text{Реш. } x' = \frac{x}{y}; \quad \frac{dx}{dy} = \frac{x}{y}, \quad \frac{dx}{x} = \frac{dy}{y}$$

$$\ln|x| = \ln|y| + c, \quad x = Cy,$$

$$y(C'y + Cy) - Cy = y^2,$$

$$\frac{dC}{dy} = 1, \quad C = y + \tilde{C}; \quad \begin{cases} x = (y + \tilde{C})y \\ y = 0 \end{cases}$$

§150

$$(1 - 2xy)y' = y(y-1);$$

$$1 - 2xy = x'y(y-1),$$

$$\frac{dx}{dy} y(y-1) = -2xy, \quad \frac{dx}{x} = -\frac{2y dy}{y(y-1)} \Rightarrow$$

$$\Rightarrow \ln|x| = -2\ln|y-1| + c,$$

$$x = \frac{C}{(y-1)^2}, \quad C(y); \quad x' = \frac{C'(y-1)^2 + C \cdot 2(y-1)}{(y-1)^4}$$

$$= \frac{C'(y-1) + 2C}{(y-1)^3} = \frac{C'}{(y-1)^2} - \frac{2C}{(y-1)^3}$$

$$\left(\frac{C'}{(y-1)^2} - \frac{2C}{(y-1)^3} \right) y(y-1) + 2 \frac{C}{(y-1)^2} y = 1$$

$$(C' - 2C)y + 2Cy(y-1)^{-1}$$

$$C'y = (y-1)^2, \quad C = \frac{(y-1)^2}{y} dy$$

$$y = 0$$

$$y = 1$$

$$\begin{cases} x = \frac{y - \ln|y| + \tilde{C}}{(y-1)^2} \\ y = 0, \quad y = 1. \end{cases}$$

7p
115

$$x-y-1+(y-x+2)y' = 0,$$

$$dx(x-y-1) + dy(y-x+2) = 0$$

$$\begin{cases} x-y=1 \\ y-x=-2 \end{cases} \Rightarrow 0 \neq -1 \Rightarrow$$

$$\Rightarrow x-y-1=u; y=x-u-1$$

$$u dx + (dx - du)(x-u-1-x+2) = 0$$

$$dx(1-u) = du(1-u)$$

$$\int dx = \int (1-u) du$$

$$x = u - \frac{u^2}{2} + c,$$

$$x = x-y-1 - \frac{1}{2}(x^2+y^2+1-2xy)$$

$$= x-y-1 - \frac{1}{2}x^2 - \frac{1}{2}y^2 - \frac{1}{2} + xy + x + y = x$$

$$x^2 + y^2 + 3 - 2xy - 2x + 2 = 0.$$

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$$(y+2)dx + (4-y-2x)dy = 0$$

$$\begin{cases} y+2=0 \\ -y-2x=-4 \end{cases} \Rightarrow -4 = 2-2x, +2 = -1+x, x=3$$

$$\begin{cases} x=3+y \\ y=-2+v \end{cases} \Rightarrow dx=du, dy=dv$$

$$v du + (4+2-v-6-2u)dv = 0$$

$$v = tu, dv = t du + u dt$$

$$t u du + (-t u - 2u)(t du + u dt) = 0$$

$$u \neq 0 \Rightarrow t du + (-2-t)(t du + u dt) = 0$$

$$du(t-2t-t^2) = (2+t)u dt$$

$$\frac{du}{u} = \frac{(2+t) dt}{t^2-t}$$

$$= -t(t+1)$$

$$\frac{2+t}{t(t+1)} = \frac{A}{t} + \frac{B}{t+1}$$

$$A=2, B=-1$$

$$\ln|u| = -2\ln|t| + \ln|t+1| + c$$

$$\ln|u| = \ln|t|^2 + \ln|t+1| + c$$

$$\ln|u| = \ln|t||t+1| + c$$

$$u = \frac{t+1}{t^2} c, c > 0, c < -1$$

122

$$2xy' + y = y^2 \sqrt{x-x^2} \Rightarrow y = z^m$$

$$2xm \cdot z^{m-1} + z^m = z^m \sqrt{x-x^2}$$

$$2m+2=1, m=-\frac{1}{2}$$

$$-xz^{-\frac{1}{2}}z' + z^{-\frac{1}{2}} = z^{-1} \sqrt{x-x^2}$$

$$z' = tx + t, -t \frac{1}{x^{\frac{3}{2}}} (tx+t) + t^{-\frac{1}{2}} = x^{-\frac{1}{2}} \sqrt{x-x^2}$$

$$-tx = t + t = \sqrt{t-1}, -tx = \sqrt{t-1}$$

$$\frac{dx}{x} = \frac{1}{x} \sqrt{t-1} = \ln|x| + c$$

$$-2\sqrt{\frac{1}{x}-1} = \ln|x|$$

140

$$x^2 y' + xy + 1 = 0, y' + \frac{y}{x} + \frac{1}{x^2} = 0$$

$$y' + \frac{1}{x}y = -\frac{1}{x^2}$$

$$\text{Hep. Aug: } \frac{dy}{y} = -\frac{1}{x} dx \Rightarrow \ln|y| = -\ln|x| + c$$

$$y = \frac{c}{x}$$

$$x^2 \frac{c}{x^2} + x \frac{-c}{x^2} + 1 = 0$$

$$c'x = -1, \int c' = \int \frac{1}{x} = c = -\ln|x| + \tilde{c}$$

$$\Rightarrow \text{Es sein: } \frac{1}{x}(-\ln|x| + \tilde{c}) = y, yx = -\ln|x| + \tilde{c}$$

141

141

$$y = x(y' - x \cos x);$$

$$y' - \frac{1}{x}y = x^2 \cos x;$$

$$\frac{dy}{y} = \frac{1}{x} dx, \ln|y| = \ln|x| + C$$

$$y = Cx, C > 0, C = \pm 1$$

$$Cx = x(C' + C - x \cos x),$$

$$x^2 \cos x = xC', \text{ for } x \neq 0: x \sin x + xC' = y$$

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$$xy' + (x+1)y = 3xe^{-x};$$

$$y' + \frac{x+1}{x}y = 3xe^{-x};$$

$$\frac{dy}{y} = -\frac{x+1}{x} dx = -(1 + \frac{1}{x}) dx$$

$$\ln|y| = -x - \ln|x| + C,$$

$$y = C e^{-x-x \ln|x|}, C > 0, C = \pm 1;$$

$$y = \frac{C}{x} e^{-x};$$

$$y' = \frac{C' - C}{x^2} e^{-x} - \frac{C}{x} e^{-x}, \frac{C' - C}{x^2} e^{-x} - \frac{C}{x} e^{-x} = 3x e^{-x}$$

$$= C' e^{-x} - \frac{C}{x} e^{-x} - C e^{-x} + \frac{C}{x} e^{-x} = 3x e^{-x} \quad \Delta 160 \quad u = \frac{1}{x};$$

$$C' = 3x^2, C = x^3 + \tilde{C}; y = \frac{x^3 + \tilde{C}}{x} e^{-x}.$$

$$y = (x^2 + C) e^{-x}.$$

157

$$(\sin y + x \cos y) y' = 1,$$

$$(\sin y + x \cos y) y' = x'$$

$$x' - \cos y x = \sin y; \quad \ln|\sin y|$$

$$\int \frac{dx}{x} = \int \cos y dy \quad \ln|x| = \sin y + C$$

$$x = C \sin y, C > 0, C = \pm 1;$$

$$C' \sin y + C \cos y = \sin y$$

$$C' \sin y + C \cos y - \sin y = 0$$

$$C' = \frac{\sin y}{\sin y}, C = \cos y + \tilde{C}$$

$$\text{Answer: } x = (-\cos y + \tilde{C}) \sin y$$

158

158. pos.

153

$$y' = y^4 \cos x + y \tan x \quad - \text{Bernoulli}$$

$$y(x) = 0 \text{ rem.}$$

$$\frac{y'}{y^4} = \cos x + \frac{\tan x}{y^3}; \quad u(x) = \frac{1}{y^3}$$

$$-3 \frac{y'}{y^5} = \cos x + \frac{\tan x}{y^3} \quad y' = \frac{u' y^4}{4};$$

$$- \frac{du}{dx} = 3 \tan x \cos x$$

$$u' + 3 \tan x u = 3 \cos x;$$

$$\frac{du}{u} = \int -3 \tan x dx; \quad \ln|u| = +3 \ln|\cos x| + C$$

$$u = C \cos^3 x; \quad C' \cos^3 x + 3C \cos^2 x \sin x + \frac{3C \cos^2 x}{\sin x} = 3 \cos x$$

$$C' = -\frac{3}{\cos^2 x}; \quad C = -3 \tan x + \tilde{C};$$

$$u = \left[-3 \tan x \sin x + C \cos^3 x = \frac{1}{y^3} \right]; y \neq 0$$

155

$$xydy = (y^2 + x) dx; \quad ydy = d\left(\frac{y^2}{2}\right)$$

$$xdu = (2u + x) dx;$$

$$xu' = 2u + x, \quad xu' = 2u, \quad \frac{1}{u} du = \frac{2}{x} dx$$

$$\ln|u| = 2 \ln|x| + C, \quad u = Cx^2$$

$$u' = 2xC + C'x^2;$$

$$x(2xC + C'x^2) = 2Cx^2 + x,$$

$$Cx^3 = x \Rightarrow C' = \frac{1}{x^2} dx, \quad C = -\frac{1}{x} + \tilde{C}.$$

$$\frac{y^2}{2} = -x + Cx^2$$

163

$$x(e^y - y)/2, \quad u = e^y, \quad u' = e^y y';$$

$$y' = \frac{u'}{u} = \frac{u'}{u^2}$$

$$x(u - \frac{u'}{u}) = 2$$

$$x(1 - \frac{u'}{u^2}) = \frac{2}{u}; \quad \frac{1}{u} = \frac{1}{e^y}; \quad 2 = \frac{1}{u^2} u'$$

$$u' = -u^2$$

$$-z'x = x - 2z; \text{ Ограничение: } \dots$$

$$x' = x - 2z$$

171

$$y' + 2ye^x - y^2 = e^{2x} + e^x \text{ Берем } a=1$$

$$y = ae^x$$

$$ae^x + 2ae^{2x} - a^2e^{2x} = e^{2x} + e^x$$

$$\begin{cases} 2a - a^2 = 1, a = 1 \Rightarrow y = u(x) + e^x \\ a = 1 \end{cases}$$

$$u' + e^x + 2(u + e^x)e^x - u^2 - 2ue^x - e^{2x} = e^{2x} + e^x$$

$$u' - u^2 = 0, \frac{du}{u^2} = dx, -\frac{1}{u} = x + C$$

$$u = -\frac{1}{x+C}$$

$$y = -\frac{1}{x+C} + e^x$$

$$y = e^x$$

169

$$xy' - (2x+1)y + y^2 = -x^2$$

$$y = ax + b, ax - (2x+1)(ax+b) + a^2x^2 + 2abx + b^2 = -x^2$$

$$\begin{cases} b + b^2 = 0 \Rightarrow b = 0, b = -1 \\ 2ab + 2ab = 0; ab - (b+1) = 0 \\ -2a + a^2 = -1, a = \pm 1 \end{cases}$$

$$a^2 - 2a + 1 = 0, (a-1)^2 = 0 \Rightarrow a = 1$$

$$y = u(x) + x$$

$$y' = u' + 1, x(u' + 1) - (2x+1)(u+x) + (u+x)^2 = -x^2$$

$$= -x^2; xu' + x - 2ux - 2x^2 - u - x + u^2 + 2ux + x^2 = -x^2$$

$$xu' - u + u^2 = 0, u = 0 \Rightarrow y = x$$

$$x \frac{u'}{u^2} - \frac{1}{u} + 1 = 0$$

$$z = \frac{1}{u}; z' = -\frac{1}{u^2} u'; \frac{1}{z^2} \frac{z'}{z^2} = -z'$$

...

$$1 - z = \frac{c}{x} \Rightarrow z = 1 - \frac{c}{x}$$

162

$$(x+1)(yy' - 1) = y^2; u = y^2, u' = 2yy'$$

$$(x+1)(u' - 2) = 2u, u' - \frac{2u}{1+x} = 2; u' - \frac{2u}{1+x} = 0$$

$$\frac{du}{u} = 2 \frac{dx}{1+x}; u = C(1+x)^2$$

$$C'(1+x)^2 + 2C(1+x) - \frac{2C'(1+x)^2}{1+x} = 2$$

$$C = -\frac{2}{1+x} + \tilde{C}; u = (1+x)^2 \tilde{C} - (1+x)2$$

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$$C = -\frac{2}{1+x} + \tilde{C}; u = (1+x)^2 \tilde{C} - (1+x)2$$

$$\frac{du}{u} = 2dy, \ln|u| = 2y + c, \\ y = \frac{1}{2} \ln|u| + c, \\ u = ce^{2y}, c > 0, \\ u' = c'e^{2y} + ce^{2y} \cdot 2 \\ c'e^{2y} + 2ce^{2y} = -2y + 1 \\ c' = -\frac{2y+1}{e^{2y}} \Rightarrow c = (1+2y)e^{-2y} + \tilde{c} \\ u = (1+2y) + \tilde{c}e^{2y} = \frac{x^2}{2};$$

Kn f.

186

$$du(x,y) = 2xydx + (x^2 - y^2)dy = 0, \\ \frac{\partial u}{\partial y} = \frac{\partial u}{\partial x}, 2x = 2x \\ \frac{\partial u}{\partial x} = 2xy, u(x,y) = \int 2xy dx + \varphi(y) = \\ = x^2 y + \varphi(y), \frac{\partial u}{\partial y} = x^2 - y^2 = x^2 + \varphi'(y) \\ \varphi'(y) = -y^2, \varphi(y) = -\frac{y^3}{3} \\ x^2 y - \frac{y^3}{3} = c,$$

189

$$\frac{y}{x} dx + (y^3 + \ln(x)) dy = 0; \\ \frac{\partial u}{\partial x} = \frac{y}{x}, u(x,y) = y \ln|x| + \varphi(y) \\ \frac{\partial u}{\partial y} = y^3 + \ln(x) = \ln|x| + \varphi'(y) \\ \Rightarrow \varphi(y) = \frac{y^4}{4} \Rightarrow u = y \ln|x| + \frac{y^4}{4}$$

201

$$(x^2 + 3 \ln y) y dx - x dy = 0 \\ (x^2 + 3 \ln y) dx - x \frac{dy}{y} = 0 \\ u = \ln y, (x^2 + 3u) dx - x du = 0$$

$$xu' - 3u = x^2, u' - \frac{3}{x}u = x^2, \\ \frac{du}{u} = \frac{3}{x} dx + \ln|u| = 3 \ln|x| \\ u = c x^3, \\ u' = 3cx^2 + 3cx^2, \\ c'x^3 + 3cx^2 - 3cx^2 = x^2, c'x = 1, \\ c = \ln|x| + \tilde{c} \\ \ln y = (\ln|x| + \tilde{c}) x^3 \\ x=0 \text{ (Bsp.)}$$

217

$$(2x^3 y^2 - y) dx + (2x^2 y^3 - x) dy = 0 \\ 2x^3 y^2 dx - y dx + 2x^2 y^3 dy - x dy = 0 \\ -d(xy) + 2x^2 y^3 dy + 2x^2 y^2 dx = 0 \\ -du + 2xu^2 dx + 2yu^2 dy = 0 \\ + \frac{du}{u^2} = 2(x^2 + y^2) d(x^2 + y^2), \\ d\left(\frac{1}{u} + x^2 + y^2\right) = 0, \frac{1}{xy} + x^2 + y^2 = \text{const.} \\ x=0, y=0$$

195

$$(x^2 + y^2 + x) dx + y dy = 0 \\ x^2 dx + y^2 dx + x dx + y dy = 0, \\ \frac{dx^2}{2} + \frac{dy^2}{2} \\ x^2 + y^2 = u, \\ u dx + \frac{du}{2} = 0, 2dx + \frac{du}{u} = 0 \\ d(2x + \ln u) = 0, \\ T. \text{ bei } u=0 \text{ unbestimmt}$$

197

$$y dy = (x dy + y dx) \sqrt{1+y^2} \\ y dy = d(xy) \sqrt{1+y^2}, \frac{y}{\sqrt{1+y^2}} dy = d(xy) \\ \frac{1}{2} \frac{dy^2}{1+y^2} = d(xy), d(\sqrt{1+y^2} - xy) = 0 \\ \Rightarrow \sqrt{1+y^2} - xy = \text{const.}$$

202

$$y^2 dx + (xy + \lg(xy)) dy = 0 \\ xy = u, y^2 dx + u dy + \lg u dy = 0 \\ x = \frac{u}{y}, dx = \frac{y du - u dy}{y^2}, \frac{1}{y^2} (y du - u dy) + \\ + u dy + \lg u dy = 0, \frac{du}{u} + \lg u = -\frac{1}{y} \\ \ln|\sin u| = -\ln|y| + c \\ \sin u = \frac{c}{u}, \text{ gain}(xy) = c$$

186

$$(x^2 + y^2 + y)dx - xy dy = 0$$

$$x^2 dx + y^2 dx + y dx - x dy = 0 \quad (y^2)$$

$$\frac{x^2}{y} dx + dx + \frac{y}{y^2} dy = 0$$

$$u = \frac{x}{y}, \quad x = uy, \quad dx = u dy + y du$$

$$u^2 dx + dx + du = 0, \quad dx = -\frac{du}{u^2 + 1}$$

$$d(x + \arctan u) = 0,$$

$$x + \arctan \frac{x}{y} = \text{const}$$

200

$$(y - \frac{1}{x}) dx + \frac{dy}{y} = 0,$$

$$y dx - \frac{1}{x} dx + \frac{dy}{y} = 0$$

$$xy dx - dx + \frac{x}{y} dy = 0, \quad \frac{xy dx}{y} + \frac{\frac{x}{y} dy - y dx}{y^2} = 0$$

$$d(\frac{x^2}{2}) - d(\frac{x}{y}) = 0$$

$$\Rightarrow x^2 - 2\frac{x}{y} = \text{const}$$

207

$$y^2 dx + (e^x - y) dy = 0,$$

$$t = e^x, \quad dt = e^x dx, \quad dx = e^{-x} dt, \quad dx = \frac{1}{t} dt$$

$$y^2 \frac{1}{t} dt + (1 - y) dy = 0,$$

$$\frac{y^2}{t} dt + t dy - y dy = 0,$$

$$\frac{y^2}{t} dt + dy - \frac{y}{t} dy = 0$$

$$\frac{y^2}{t} dt + t dy - dy = 0$$

$$y^2 \cdot \frac{dt}{t^2} - y \cdot \frac{1}{t} dy + dy = 0,$$

$$y^2 \frac{t'}{t^2} - y \cdot \frac{1}{t} + 1 = 0, \quad u = \frac{1}{t},$$

$$-y^2 \cdot u' - y \cdot u + 1 = 0,$$

$$y^2 u' + y \cdot u = 1, \quad y^2 u' + y \cdot u = 0,$$

$$y \cdot u' + u = 0,$$

$$y \frac{du}{dy} = -u \Rightarrow$$

$$\ln u = -\ln y + C, \quad u = \frac{C}{y}, \quad u' = \frac{y' - C}{y^2}$$

$$ye' - e + e = 1, \quad e' = \frac{1}{y}, \quad e = \ln|y| + C$$

$$\Rightarrow \int \ln|y| + C = \frac{y}{e^x}$$

188

$$e^{-y} dx - (2y + x e^{-y}) dy = 0,$$

$$-e^{-y} = -e^{-y}$$

$$u(x, y) = \int e^{-y} dx + \varphi(y) = e^{-y} x + \varphi(y)$$

$$\frac{\partial u}{\partial y} = -(2y + x e^{-y}) = x \cdot e^{-y} (-1) + \varphi'(y)$$

$$\varphi'(y) = -y^2 + C, \quad e^{-y} x - y^2 = C$$

192

$$(1 + y^2 \sin^2 x) dx - 2y \cos^2 x dy = 0$$

$$2y \sin 2x + 2y (2 \sin x) \cos x$$

$$u(x, y) = \int (1 + y^2 \sin^2 x) dx + \varphi(y)$$

$$x + y^2 \cos^2 x + \varphi = \text{const}$$

$$\frac{\partial u}{\partial y} = -2y \cos^2 x \Rightarrow \varphi = -\frac{y^2}{2}$$

$$x - \frac{y^2}{2} \cos^2 x - \frac{y^2}{2} = \text{const}$$

212, 215, 220, 221, 204, 208, 195, 206, 207

2/p

211

$$(2x^2 y^2 + y) dx + (x^2 y - x) dy = 0 \quad x_y' = 2x$$

$$\frac{\partial^2 u}{\partial x \partial y} = 4xy + 1 \neq \frac{\partial^2 u}{\partial y \partial x} = 3x^2 - 1$$

$$2x^2 y^2 dx + y dx + x^2 y dy - x dy = 0$$

$$xy(y dx^2 + x^2 dy) + y dx - x dy = 0 \quad | xy \neq 0$$

$$d(x^2 y) + \frac{dx}{x} + \frac{dy}{y} = 0$$

$$d(x^2 y) + d(\ln x) + d(\ln y) = 0,$$

$$d(x^2 y) + d(\ln x - \ln y) = 0 \Rightarrow$$

$$x^2 y + \ln \frac{x}{y} = C, \quad x=0, y=0.$$

215

$$x(\ln y + 2 \ln x - 1) dy = 2y dx$$

$$x \ln y dy + 2x \ln x dy - x dy = 2y dx$$

$$\begin{aligned}
 -2y^2 d\left(\frac{\ln x}{y}\right) + (\ln y - 1) \cdot dy &= 0 \\
 \ln y dy + 2 \ln x dy - dy &= 2y d \ln x \\
 2(\ln x dy - y d \ln x) + \ln y dy - y d \ln y &= 0 \\
 -2y^2 d\left(\frac{\ln x}{y}\right) - y^2 d\left(\frac{\ln y}{y}\right) &= 0 \quad | : y^2 \neq 0 \\
 2d\left(\frac{\ln x}{y}\right) + d\left(\frac{\ln y}{y}\right) &= 0 \\
 \ln x^2 y = y \cdot C, \quad y = C \ln x^2 y, \\
 x=0, y=0.
 \end{aligned}$$

220

$$\begin{aligned}
 y^2(y dx - 2x dy) &= x^3(x dy - 2y dx) \\
 (y^3 + 2yx^3) dx &= (x^4 - 2xy^2) dy \\
 \frac{y^2 dx - 2xy dy}{x^2} &= y \cdot \frac{x^2 dy - 2xy dx}{y^2} \\
 -d\frac{y^2}{x} &= -y \cdot d\frac{x^2}{y}, \quad \frac{y^2}{x} = u, x = \frac{y^2}{u} \\
 \frac{x^2}{y} &= v, y = \frac{x^2}{v} = \frac{y^4}{u^2 v}
 \end{aligned}$$

$$\begin{aligned}
 y^3 &= u^2 v \Rightarrow y = u^{\frac{2}{3}} v^{\frac{1}{3}} \\
 du &= y dv, \quad du = u^{\frac{2}{3}} v^{\frac{1}{3}} dv \\
 \frac{du}{u^{\frac{2}{3}}} &= v^{\frac{1}{3}} dv \\
 3u^{\frac{1}{3}} &= \frac{3}{4} v^{\frac{4}{3}} + C
 \end{aligned}$$

$$\begin{aligned}
 3\left(\frac{y^2}{x}\right)^{\frac{1}{3}} &= \frac{3}{4}\left(\frac{x^2}{y}\right)^{\frac{4}{3}} + C, \quad 3y^2 - \frac{3}{4}x^2 = 4y^{\frac{1}{3}}x^{\frac{2}{3}} \\
 x^3 - 4y^2 &= 2y^{\frac{1}{3}}x^{\frac{2}{3}}.
 \end{aligned}$$

$$x=0, y=0; \quad x^3 - 4y^2 = 2y^{\frac{1}{3}}x^{\frac{2}{3}}.$$

198

$$\begin{aligned}
 xy^2(xy' + y) &= 1, \quad xy^2 dy + xy^3 dx = dx \\
 xy^2 dy + (xy^3 + 1) dx &= 0 \\
 \frac{1}{3}x^2 d\frac{y^3}{x} + \frac{1}{2}(x^2 y^3)' dx &= 0; \\
 (xy)^2 (xy)' &= x, \quad xy = u, \quad u^2 u' = x \\
 \frac{1}{3}(u^3)' &= x \\
 u^3 &= \frac{3}{2}x^2 + C, \quad 2x^2 y^3 - 3x^2 = C.
 \end{aligned}$$

204

$$\begin{aligned}
 y(y^2 + 1) dx + x(y^2 - x + 1) dy &= 0 \\
 y dx + x\left(1 - \frac{x}{y^2 + 1}\right) dy &= 0,
 \end{aligned}$$

$$\begin{aligned}
 y^3 dx + y dx + xy^2 dy - x^2 dy + x dy &= 0 \\
 (y^2 + 1)(y dx + x dy) - x^2 dy &= 0; \\
 \frac{y dx + x dy}{x^2 y^2} - \frac{dy}{y^2(y^2 + 1)} &= 0
 \end{aligned}$$

$$\frac{d(xy)}{(xy)^2} = \frac{dy}{y^2(y^2+1)}, \quad \frac{du}{u^2} = \frac{dy}{y^2(y^2+1)}, \quad u = xy$$

$$\int \frac{du}{u^2} = \int \left(\frac{1}{y^2} - \frac{1}{y^2+1}\right) dy + C,$$

$$\frac{x-y}{x^2 y^2} + \arctan y = C.$$

209

$$x^2 y(y dx + x dy) = 2y dx + x dy \quad | : x^2 y$$

$$d(xy) = 2 \frac{dx}{x^2} + \frac{dy}{xy}, \quad u = xy$$

$$du = -\frac{2}{x^3} dx + \frac{dy}{u}, \quad y = \frac{u}{x},$$

$$(x^2 y - 1) d(xy) = y dx, \quad dy = -\frac{u}{x^2} dx$$

$$(x^2 y - 1) \frac{d(xy)}{xy} = \frac{dx}{x}, \quad u = xy$$

$$(xu - 1) \frac{du}{u} = \frac{dx}{x}, \quad (u - \frac{1}{x}) \frac{du}{u} = \frac{dx}{x^2},$$

$$-\frac{1}{x} = v, \quad (u+v) \frac{du}{u} = dv = 0,$$

$$u du + v du - u dv = 0$$

$$\frac{du}{u} - d\left(\frac{v}{u}\right) = 0, \quad \ln|u| - \frac{v}{u} = \text{const}$$

$$x^2 y \ln(xy) = -1.$$

199

$$y^2 dx - (xy + x^3) dy = 0$$

$$\frac{1}{2} y^2 d\left(\frac{1}{x^2}\right) - \frac{1}{2} x^2 dy^2 - dy = 0$$

$$-\frac{1}{2} \left(d\left(\frac{y^2}{x^2}\right)\right) - dy = 0, \quad d\left(-y - \frac{1}{2} \frac{y^2}{x^2}\right) = 0$$

$$y + \frac{1}{2} \frac{y^2}{x^2} = C, \quad 2y + y^2 = Cx^2, \quad x=0.$$

206

$$y dx - x dy = 2x^2 \tan \frac{y}{x} dx$$

$$d\left(\frac{y}{x}\right) = \tan \frac{y}{x} dx, \quad u = \frac{y}{x},$$

$$du = \left(\tan u\right) \frac{2x+1}{x^2} dx \Rightarrow \frac{du}{\tan u} = 2x dx$$

$$\Rightarrow \left|\ln \frac{2}{x}\right| = C e^{x^2}.$$

2.272
 $y' dx + (e^x - y) dy = 0$

h. p. metoda

2.273
 $y' y^2 = 2 = y$, $y' = \frac{1}{2} y^{\frac{1}{2}}$, $\frac{dy}{y^{\frac{1}{2}}} = \frac{1}{2} dx$
 $\Rightarrow \dots y^2 = (x-c)^3, y=0$

2.275
 $y'' + y' = y y' (y' + 1)$
 $y'' - y' y' = y y' + y' = 0$
 $(y')^2 (y' - y) + y' (y - y') = 0$
 $(y')^2 (y' - y) = 0$
 1) $y' = -y$ 2) $y' = y \Rightarrow$
 $\Rightarrow y = (x^2 + c)^{-1}$ but $|y| = x + c$
 $y = c \cdot e^x$
 $2y' = x + c$

2.277
 $x = (y')^2 = y'$, $y' = p$, $dy = p dx$
 $x = p^3 + p$, $dx = 3p^2 dp + dp$
 $\frac{dx}{p^3} = (3p^2 + 1) dp$
 $dy = p(3p^2 + 1) dp$
 $\begin{cases} y = \frac{3}{4} p^4 + \frac{1}{2} p^2 + c \\ x = p^3 + p \end{cases}$

2.280
 $y'(e - \ln y) = 1$
 $p = y'$ $dy = p dx$
 $p(e - \ln p) = 1$
 $x = \frac{1}{p} + \ln p$

$\frac{dx}{p} = -\frac{dp}{p^2} + \frac{dp}{p}$
 $\frac{dy}{p} = -\frac{1}{p^2} dp + \frac{1}{p} dp$
 $dy = -\frac{1}{p} dp + dp$
 $y + c = -\ln p + p$

$\begin{cases} y = -\ln p + p + c \\ x = \frac{1}{p} + \ln p \end{cases}$

2.271
 $y = y^2 + 2y^3$, $y' = p$, $dy = p dx$
 $y = p^2 + 2p^3$, $dy = 2p dp + 6p^2 dp$
 $p dx = 2p dp + 6p^2 dp =$
 $= dp(2p + 6p^2)$, $p dx = (2 + 6p) dp$
 $\begin{cases} x = 2p + 3p^2 + c \\ y = p^2 + 2p^3, y=0 \end{cases}$

2.271
 $(y')^3 + y^2 = x y y'$ $y' = p$
 $p^3 + y^2 = x y p$, $x = \frac{1}{y} p^2 + \frac{1}{p} y$
 $dx = \frac{d}{dy} \left(\frac{1}{y} p^2 + \frac{1}{p} y \right) dy = \left(-\frac{p^2}{y^2} + \frac{1}{p} \right) dy + \left(\frac{2p}{y} - y p^{-2} \right) dp$
 $dx = \frac{dx}{p}$; $dy = \left(\frac{2p^2}{y} - y p \right) dp + \left(-\frac{p^2}{y^2} + \frac{1}{p} \right) dy$
 $(2p^3 y - y^3) dp - p^3 dy = 0$
 $p^4 y' - 2p^3 y - y^3 = 0$
 $\frac{p^4}{y^3} y' - \frac{2p^3}{y} = -1 = 0$, $z = \frac{1}{y^2}$
 $z' = -2 \frac{y'}{y^3}$, $-\frac{p^3}{2} z' - 2p^3 z + 1 = 0$
 $p^3 z' + 4p^3 z = 0$
 $\frac{dz}{z} = -4 \frac{dp}{p}$, but $|z| = -4 \ln |p| + c$
 $z = \frac{c}{p^4}$
 $z = \frac{c' p^4}{p^4} = c' \Rightarrow \dots$

$\begin{cases} y^2 = \frac{1}{2p+c} \\ x = \frac{1}{y} + \frac{1}{p} \\ y=0 \end{cases}$

2.278

$$(y')^2 - 2xy' = x^2 - 4y$$

$$y' = p, \quad p^2 - 2xp = x^2 - 4y$$

$$x^2 + 2xp - (4y + p^2) = 0$$

$$4y = x^2 - p^2 + 2xp$$

$$4y' dy = 2x dx + (-2p + 2x) dp$$

$$4p dx = (2x + 2p) dx + (2x - 2p) dp$$

$$(x - p) dx = (p - x) dp$$

$$\begin{cases} x = p \rightarrow y = 2x^2 \\ dx = -dp \rightarrow x = C - p, p = C - x \end{cases}$$

$$4y = x^2 - (C-x)^2 + 2(C-x)(C-x)$$

282

$$y = xy' - (y')^2, \quad y' = p, \quad dy = p dx$$

$$y = xp - p^2, \quad dy = p dx + (x - 2p) dp$$

$$p dx + (x - 2p) dp = 0$$

$$\begin{cases} x = 2p \rightarrow y = \frac{x^2}{2} - \frac{x^2}{4} = \frac{x^2}{4} \\ dp = 0 \end{cases}$$

$$p = \text{const}$$

$$y = xe^{-p^2}$$

289

$$y = 2xy' - 4(y')^3, \quad y' = p$$

$$y = 2xp - 4p^3 \quad | d$$

$$dy = 2x dp + (2p - 12p^2) dp$$

$$p dx, \quad dx(-p) = (2x - 12p^2) dp$$

$$p \frac{dx}{dp} + 2x = 12p^2$$

$$\frac{dx}{x} = 2 dp, \quad \ln|x| = 2p + C$$

$$x = ce^{2p}$$

$$p \frac{dx}{dp} + 2x = 0, \quad \frac{dx}{x} = -2 dp$$

$$x = \frac{C}{p^2}$$

253

$$xy' - 2yy' + x = 0, \quad y' = p$$

$$xp^2 - 2yp + x = 0$$

$$y = \frac{1}{2}xp + \frac{1}{2}p^2x, \quad p dx = (\frac{1}{2}p + \frac{1}{2}p) dx + (\frac{1}{2}x + \frac{1}{2}p^2x) dp$$

$$2p^3 dx = (p^3 + p^3) dx + (xp^2 - x) dp$$

$$0 = (p^3 - p^3) dx + x(p^2 - 1) dp$$

$$-p^3(p^2 - 1) \Rightarrow (p^2 - 1)(p dx - x dp) = 0$$

$$\Rightarrow p = \pm 1 \Rightarrow y = \pm x$$

$$\frac{dx}{x} = \frac{dp}{p} \Rightarrow \ln|x| = \ln|p| + C$$

24

243

$$(y' + 1)^3 = 27(x + y)^2, \quad t = x + y$$

$$\frac{dt}{dx} = y' + 1$$

$$(t')^3 = 27(t)^2$$

$$t' = 3t^{\frac{2}{3}}, \quad \int t^{-\frac{2}{3}} = \int 3 dx$$

$$\frac{t^{\frac{1}{3}}}{\frac{1}{3}} = 3x + C$$

$$\left[\frac{1}{3} = 3(x + C)(x + y)^{\frac{1}{3}} \rightarrow (x + y)^{\frac{1}{3}} = x + C \right]$$

$$y = -x$$

247

$$xy'^2 = y, \quad y'^2 = \frac{y}{x^2}, \quad \int \frac{dy}{y^{\frac{1}{2}}} = \int \frac{dx}{x^{\frac{1}{2}}}$$

$$\frac{1}{\frac{1}{2}} y^{\frac{1}{2}} = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$y = \frac{1}{4}(x + C)^2$$

2952

$$xy'(xy' + y) = 2y^2, \quad y' = t$$

$$(y^2 x)'; \quad (t \cdot y)(t \cdot x) = 2y^2$$

$$v(x + y) = 2y^2 y'$$

$$x'(y')^2 + xy'y' - 2y^2 = 0$$

$$1) = x^2 y' + 8x^2 y' = 9x^2 y'$$

$$y' = \frac{-x \pm \sqrt{x^2 + 4x}}{2x^2}, \quad y_1' = -\frac{2}{x}, \quad y_2' = \frac{2}{x}$$

$$1) \frac{dy}{dx} = \frac{2}{x}, \quad dy = \frac{dx}{x}, \quad y = Cx,$$

$$2) \frac{dy}{dx} = -\frac{2}{x}, \quad dy = -2 \frac{dx}{x}, \\ \text{dividing: } yx^2 = C, \quad y = Cx$$

255

$$y'^2 + x = 2y, \quad y'(y'-1)^2 - 1 + x = 0, \\ y'^2 - 1 = 1 - x \\ y' - 1 = \pm(1-x)$$

$$y' = \frac{\pm(1-x)+1}{1} \\ \frac{dy}{dx} = 1$$

268

$$x(y'^2 - 1) = 2y$$

$$x(p^2 - 1) = 2y, \quad x = \frac{2y}{p^2 - 1}, \quad dx = \frac{2p^2 - 2 - 2p dp}{(p^2 - 1)^2} \\ \frac{dy}{p}$$

$$\Rightarrow y = -\ln|p^2 - 1| + \frac{C}{p^2 - 1} + C \\ x = \frac{2y}{p^2 - 1}$$

269

$$x = y' \sqrt{y'^2 + 1}, \quad y' = p, \quad p dx = dy$$

$$x = p \sqrt{p^2 + 1}$$

$$dx = (\sqrt{p^2 + 1} + p \cdot \frac{p dp}{\sqrt{p^2 + 1}}) dp$$

$$dy = p \cdot \frac{2p^2 + 1}{\sqrt{p^2 + 1}} dp$$

$$\int \frac{2p^2 + 1}{\sqrt{p^2 + 1}} p dp = 2 \int \frac{p^2 + 1}{\sqrt{p^2 + 1}} dp - \int \frac{p dp}{\sqrt{p^2 + 1}}$$

$$= 2 \int \sqrt{p^2 + 1} dp - \int \frac{p dp}{\sqrt{p^2 + 1}} = \frac{2}{3} (p^2 + 1)^{3/2} - \sqrt{p^2 + 1}$$

272

$$y = \ln(1 + (y')^2), \quad p = y'; \quad dy = p dx \\ dy = \frac{1}{1 + p^2} \cdot 2p dp, \quad dx = 2 \frac{dp}{1 + p^2}$$

$$dx = 2 d(\arctan p),$$

$$d(x - 2 \arctan p) = 0, \quad x - 2 \arctan p = C \\ (y = \ln(1 + p^2))$$

274

$$y = (y' - 1)e^{y'}, \quad y' = p, \quad dy = p dx$$

$$y = (p - 1)e^p, \quad dy = e^p(1 + p - 1) dp \\ dx = e^p dp, \quad x = e^p + C \\ y = (p - 1)e^p$$

282

$$2xy' - y = y' \ln y, \quad y' = p, \quad dy = p dx$$

$$y = 2xp - p \ln y \\ \frac{dy}{p dx} = (2p) dx + (-p \frac{1}{y} \cdot p) dy + (2x - \ln y - 1) p$$

$$2xp - y = p \ln y, \quad 2xp = y + p \ln y$$

$$x = \frac{y}{2p} + \frac{1}{2} \ln y$$

$$x = \frac{y}{2p} + \frac{1}{2} (\ln y + \ln p)$$

$$\frac{dx}{dy} = \frac{1}{2p} - \frac{y}{2p^2} \frac{dp}{dy} + \frac{1}{2} \left(\frac{1}{y} + \frac{1}{p} \frac{dp}{dy} \right)$$

$$\frac{1}{p} = \frac{1}{2p} - \frac{y}{2p^2} + \frac{1}{2y} + \frac{p'}{2p}$$

$$\frac{2 - 1 - p'}{2p} = \frac{1}{2y} - \frac{y}{2p^2} + \frac{p - p' + y p'}{2p^2} = \frac{1}{2y}$$

$$\frac{p'(y - p)}{2p^2} = \frac{1}{2y} - \frac{1}{2p}$$

$$\frac{p'(y - p)}{2p^2} = \frac{p - y}{2yp}$$

$$a) y = p, \quad x = \frac{y}{2p} + \frac{1}{2} \ln y = \frac{1}{2} + \frac{1}{2} \ln y \\ 2x = 1 + \ln y$$

$$b) \frac{p'}{p} = -\frac{1}{y}, \quad \frac{dp}{p} = -\frac{dy}{y}$$

$$p = \frac{C}{y}$$

285

$$y' = e^{xy/y}, \quad y' = p, \quad \ln p = x p / y \\ x = \frac{y}{p} \ln p$$

$$\frac{dx}{dy} = \left(\frac{\ln p}{p} \right) dy + y \cdot \frac{\frac{1}{p} - \ln p}{p^2} dp$$

$$\frac{dx}{p} \cdot \frac{dy}{y} = \frac{dy}{p} (1 - \ln p) dy = \left(\frac{1}{p} (1 - \ln p) \right) dp$$

$$\frac{dy}{y} = \frac{dp}{p}, \ln y = \ln p + C$$

$$y = Cp$$

Задача

$$y = xy' - x^2 y', \quad y' = p, \quad dy = p dx$$

$$y = xp - x^2 p^3, \quad dy = (p - 2xp^3) dx +$$

$$x(2xp^3) dp = (x - 3x^2 p^2) dp$$

$$2xp^3 = p^2 (x - 3x^2 p^2)$$

$$p' = \frac{2xp^3}{x - 3x^2 p^2} = \frac{2p^3}{1 - 3xp^2}$$

a) $p=0, \quad y = xp - x^2 p^3 = 0$

b) $p' = \frac{1}{x'}, \quad \frac{1}{x'} = \frac{2p^3}{1 - 3xp^2}$

$$x' = \frac{1 - 3xp^2}{2p^3} = \frac{1}{2p^3} - \frac{3x}{2p}$$

$$x' + \frac{3x}{2p} = \frac{1}{2p^3}$$

$$\frac{dx}{dp} = -\frac{3x}{2p}, \quad \frac{dx}{x} = -\frac{3}{2} \frac{dp}{p}$$

$$x = Cp^{-\frac{3}{2}}$$

$$C = -\frac{1}{p} + C_1$$

$$x = \left(-\frac{1}{p} + C_1 \right)^{-\frac{2}{3}}$$

Задача

$$y = xy' - (2+y')$$

$$y+2 = y'(x-1),$$

$$\frac{dy}{y+2} = \frac{dx}{x-1}; \quad y+2 = (x-1)C$$

$$y = (x-1)C - 2;$$

Задача

$$2xy' - y = \ln y'; \quad p = y', \quad dy = p dx$$

$$2xp - y = \ln p, \quad y = 2xp - \ln p$$

$$dy = 2p dx + \left(2x - \frac{1}{p} \right) dp$$

$$0 = p dx + \left(2x - \frac{1}{p} \right) dp;$$

$$2p + 2xp' - p = p'/p, \quad p = \frac{p'(1-2px)}{p}$$

$$p^2 = p'(1-2px),$$

$$p' = \frac{1}{x}; \quad x' + \frac{2}{p} x = \frac{1}{p^2}$$

$$C = p + C';$$

$$x = \frac{1}{p} + \frac{C'}{p^2}$$

$$y = 2 + \frac{2C'}{p} - \ln p$$

Кр. раб.

Задача

$$x^2 y'' = (y')^2, \quad y'(x) = u(x), \quad y''(x) = u'(x)$$

$$x^2 u' = u^2, \quad 1: u^2,$$

$$x^2 \frac{u'}{u^2} = 1, \quad \frac{1}{u} = t, \quad t' = -\frac{1}{u^2} u'$$

$$-x^2 t' = 1, \quad \frac{dy}{dx} = \frac{x}{1+x C_1}$$

$$\int dy = \int \frac{x}{1+x C_1} dx$$

1) $C_1 = 0; \quad dy = x dx, \quad y = \frac{x^2}{2} + C_2$

2) $C_1 \neq 0; \quad dy = \frac{x}{C_1 (\frac{1}{C_1} + x)} dx =$

$$= \frac{(x + \frac{1}{C_1}) - \frac{1}{C_1}}{C_1 (\frac{1}{C_1} + x)} dx = \frac{1}{C_1} dx - \frac{1}{C_1^2} \frac{d(\frac{1}{C_1} + x)}{\frac{1}{C_1} + x}$$

$$y = \frac{x}{C_1} - \frac{1}{C_1^2} \ln \left| x + \frac{1}{C_1} \right| + C_2$$

$$u = \text{const}$$

Задача

$$y''(e^x + 1) + y' = 0, \quad u = y'$$

$$u'(e^x + 1) + u = 0,$$

$$u' + \frac{1}{e^x + 1} u = 0, \quad \frac{du}{u} = -\frac{dx}{e^x + 1}$$

$$\int \frac{dx}{e^x + 1} = \int \frac{t \geq e^x}{t \geq e^x + 1} dt = \int \frac{1}{t} \frac{dt}{t+1} = \int \frac{dt}{t(t+1)}$$

$$x \geq \ln t, \quad dx = \frac{1}{t} dt = \ln e^x - \ln(e^x + 1) + C$$

$$- \ln|u| = \ln e^x - \ln(e^{x+1}) + C$$

$$\frac{1}{u} = \frac{t}{t+1} \cdot C, \quad y' = \frac{\ln(x)}{t}$$

$$dy = dx \left(C_1 + \frac{C_2}{t} \right)$$

$$y = C_1(t + \ln t) + C_2$$

$$\begin{cases} y = C_1(x - e^{-x}) + C_2 \\ u = 0 \end{cases}$$

433

$$y'^2 + y' = xy'', \quad y' = u,$$

$$u^2 + u = xu', \quad u' = p, \quad du = p dx$$

$$p^2 + u = xp, \quad u = xp - p^2,$$

$$\frac{du}{dx} = p + (x - 2p)p$$

$$1) x = 2p, \quad u = \frac{x^2}{2} - \frac{x^2}{4} = \frac{x^2}{4} = y',$$

$$y = \frac{1}{4} \frac{x^3}{3} + C$$

$$2) dp = 0, \quad p = C_1 = u', \quad u = C_1 x + C_2$$

$$u = px - p^2, \quad u = y'$$

$$y' = C_1 x - C_2^2$$

434

$$y'' + y'^2 = 2e^{-y}, \quad y'(x) = u(y)$$

$$y'' = \frac{du}{dx} = \frac{du}{dy} \cdot \frac{dy}{dx} = \frac{du}{dy} \cdot u$$

$$u(u' + u^2) = 2e^{-y}$$

$$u' + u^2 = 2e^{-y} \quad t(y) = u^2; \quad t' = 2u u'$$

$$\frac{1}{2} t' + t = 2e^{-y}, \quad t' + 2t = 4e^{-y}$$

$$\frac{dt}{t} = -2dy$$

$$t = C e^{-2y} + C$$

$$C e^{-2y} - 2C e^{-2y} + 2C e^{-2y} = 4e^{-y}$$

$$C' = 4e^{-y}, \quad C = 4e^{-y} + C_2$$

$$t = (4e^{-y} + C_2) e^{-2y}$$

$$(y')^2 = u^2 = t$$

$$\left(\frac{dy}{dx} \right)^2 = C_1 e^{-y} + 4e^{-y}$$

$$\frac{dy}{\sqrt{C_1 e^{-y} + 4e^{-y}}} = \pm dx$$

$$\int \frac{e^y dy}{\sqrt{C_1 + 4e^y}} = \pm \int dx$$

$$\frac{1}{4} \int \frac{d(e^y + C_1)}{\sqrt{C_1 + 4e^y}} = \frac{1}{4} 2\sqrt{e^y + C_1} = \pm x + C_2$$

424

$$y'^2 + 2yy'' = 0,$$

$$y'' = u'u', \quad y' = u; \quad u^2 + 2yu'u = 0$$

$$1 + 2y \frac{u'}{u} = 0, \quad 2y \frac{u'}{u} = -1, \quad \frac{u'}{u} = -\frac{1}{2y}$$

$$\int \frac{du}{u} = -\frac{1}{2} \int \frac{dy}{y}; \quad u = C e^{-\frac{1}{2}y}$$

$$y', \quad y' - C e^{-\frac{1}{2}y} = 0$$

$$\frac{dy}{y} = C e^{-\frac{1}{2}y} dy$$

$$\ln|y| = C e^{-\frac{1}{2}y} + C_2$$

428

$$y''' = y'^2, \quad y''' = u''(x), \quad u'(x) = y''$$

$$u' = u', \quad \frac{du}{u^2} = dx, \quad x + C = -\frac{1}{u}$$

$$\Rightarrow u = -\frac{1}{x+C} = y''(x); \quad \frac{d}{dx} \left(\frac{dy}{dx} \right) = -\frac{1}{x+C}; \quad \frac{dy}{dx} = -\ln|x+C| + C_1$$

$$y = C_2 x - \int \ln|x+C| dx + C_3$$

$$u = y' = C_4 x + C_5$$

462

$$xy'' - xy'^2 = y \cdot y', \quad y'(x) = y(x) \cdot z(x)$$

$$y''(x) = y'z + z'y$$

$$-y y z^2 + 2zy, \quad x(yz + z'y) - x y z^2 = y z^2$$

$$x z'y = y z^2, \quad z' - \frac{y}{x} z = 0$$

$$xz' = z, \quad z = C_1 x, \quad z = \frac{y'}{y} = C_1 x$$

$$y = C_2 e^{C_1 x^2}$$

447

$$x^2 y y' = (y - x y')^2, \quad y' = y z$$

$$y'' = y z' + z y$$

$$x^2 y (y z' + z y) = (y - x y')^2$$

$$x y^2 z' + x^2 y^2 z' = y^2 - 2 x y^2 z + x^2 y^2 z^2$$

$$\begin{cases} x^2 z' = 1 - 2 x z, & z' + \frac{z}{x} = \frac{1}{x^2} \\ y = 0 \end{cases}$$

$$\frac{dz}{z} = -\frac{1}{x} dx$$

$$z = \frac{C}{x^2}$$

$$2) z = \frac{C}{x^2}, \quad dz = \frac{-2C}{x^3} dx$$

442
445
464
466

2/p

442

$$y''(2y' + x) = 1, \quad y' = u, \quad dy = u dx$$

$$u'(2u + x) = 1, \quad \frac{du}{dx} = \frac{1}{2u + x}$$

$$2u + x = x', \quad x' - x = 2u$$

$$\frac{dx}{x} = du$$

$$\ln|x| = u + C_1$$

$$x = C_1 e^u$$

$$x' = C_1 e^u (C_1' + 1)$$

$$e^u (C_1' + 1) - C_1 e^u = 2u$$

$$C_1' = \frac{2u}{e^u}$$

$$C_1 = \int \frac{2u}{e^u} du = \int \frac{2t}{e^t} dt = 2 \int \frac{t}{e^t} dt$$

$$= 2 \int \frac{t}{e^t} dt$$

$$= \frac{u^2}{e^u} + \int e^{-u} u du, \quad C_1 = -2e^{-u}(u+1) + C_2$$

$$x = C_2 e^u - 2u - 2$$

$$dx = C_2 e^u du - 2du, \quad u = y' = \frac{dy}{dx}$$

$$2) dx = \frac{dy}{u}$$

$$\frac{dy}{u} = (C_2 e^u - 2) du, \quad dy = (C_2 e^u - 2) u du$$

$$y = \int (C_2 e^u - 2) u du$$

$$y = C_2 e^u (u-1) - u^2 + C_3$$

$$\begin{cases} x = C_2 e^u - 2u - 2 \\ y = C_2 e^u (u-1) - u^2 + C_3 \end{cases}$$

445

$$y y'' - 2 y y' \ln y = y'^2$$

$$y' = p, \quad p' p = y''$$

$$p(y p' - 2 y \ln y - p) = 0$$

$$1) p = 0, \quad y' = 0, \quad y = C$$

$$2) y p' - p = 2 y \ln y$$

$$y p' = p, \quad p = C y$$

$$C = 2 \int \frac{\ln y}{y} dy = \ln^2 y + C_1$$

$$p = C y = (\ln^2 y + C_1) y$$

$$y' = y (\ln^2 y + C_1)$$

$$\frac{dy}{dx} = y (\ln^2 y + C_1), \quad x = \int \frac{dy}{y (\ln^2 y + C_1)}$$

$$= \int \frac{t = \ln y}{t^2 + C_1} dt$$

$$1) C_1 = 0 \Rightarrow \ln y (1 - x) = 1$$

$$2) C_1 > 0: \quad x = \int \frac{dt}{t^2 + C_1} = \frac{1}{C_1} \arctan \frac{t}{C_1} + C_2$$

$$\ln y = C_1 \arctan(C_1 x + C_2)$$

$$3) C_2 < 0: \quad x = \int \frac{dt}{t^2 + C_1} = \frac{1}{C_1} \ln \left| \frac{t - C_1}{t + C_1} \right| + C_2$$

$$\ln y = \frac{1}{C_1} \ln \left| \frac{t - C_1}{t + C_1} \right| + C_2$$

464

$$y y'' = y'^2 + 15 y^2 \sqrt{x}$$

$$y' = y z, \quad y'' = y' z + y z' = y z^2 + y z'$$

$$y(yz' + yz') = y^2 z' + 15y^2 \sqrt{x} \quad (y=0)$$

$$z' = 15\sqrt{x}, \quad z = 15 \cdot 2 \cdot x^{\frac{3}{2}} + C_1 =$$

$$= 10x^{\frac{3}{2}} + C_1 \Rightarrow y' = 10x^{\frac{3}{2}}y + C_1 y,$$

$$\frac{dy}{y} = (10x^{\frac{3}{2}} + C_1) dx \Rightarrow \ln|y| = 10 \cdot \frac{2}{5} x^{\frac{5}{2}} + C_1 x + C_2$$

$$\Rightarrow y = C_2 e^{4x^{\frac{5}{2}} + C_1 x}$$

✓ 466

$$xyy'' + xy'^2 = 2yy', \quad y' = yz$$

$$yy'' = yz^2 + yz'$$

$$xy(yz^2 + yz') + x(yz^2) = 2yz^2$$

$$xyyz^2 + xyyz' + xyyz^2 = 2yz^2$$

$$xz' = 2z$$

$$z' = 2 \frac{z}{x}, \quad \frac{dz}{z} = \frac{dx}{x}, \quad \ln|z| = \ln|x| + C_1$$

$$z = C_1 \frac{x}{2}$$

$$\Rightarrow y' = C_1 \cdot \frac{y}{x}$$

$$\frac{dy}{y} = \frac{C_1}{2} dx, \quad \ln|y| = \frac{C_1}{2} x^2 + C_2$$

$$y = e^{\frac{C_1}{2} x^2 + C_2}$$

Контроль

✓ 473

$$4x^2 y'' = x^2 - y^4 \quad - \text{однородное - уравнение}$$

$$y = x^m; \quad 2 + 3m + m - 2 = 2 = 4m$$

$$m = \frac{1}{2}$$

$$\{x = e^t, y = z(t) e^{\frac{1}{2}t}\}$$

$$y'_x = \frac{dz}{dt} \frac{dt}{dx} = \frac{dz}{dt} e^{-\frac{1}{2}t} = (ze^{\frac{1}{2}t})' e^{-t} =$$

$$= (z'e^{\frac{1}{2}t} + z \cdot \frac{1}{2} e^{\frac{1}{2}t}) e^{-t} = e^{-\frac{1}{2}t} (z' + \frac{1}{2}z)$$

$$y''_{xx} = (e^{-\frac{1}{2}t} (z' + \frac{1}{2}z))' \frac{dt}{dx} =$$

$$= e^{-\frac{3}{2}t} (z'' - \frac{1}{4}z)$$

$$4e^{2t} \cdot z^3 e^{\frac{3}{2}t} \cdot e^{-\frac{3}{2}t} (z'' - \frac{1}{4}z) = e^{2t} - z^4 e^{2t}$$

$$4e^{2t} z^3 (z'' - \frac{1}{4}z) = e^{2t} (1 - z^4)$$

$$4z^3 z'' = 1, \quad z'' = \frac{1}{4z^3}, \quad z' = u(z)$$

$$z'' = u \cdot u' = \frac{1}{4z^3}, \quad \frac{du}{dz} = \frac{1}{4z^2 u}$$

$$\Rightarrow \frac{u^2}{2} = \frac{1}{4} \cdot \frac{1}{z^2} \cdot \frac{1}{(-2)} + C, \quad u^2 = -\frac{1}{4z^2} + C_1$$

$$z' = \pm \sqrt{C_1 - \frac{1}{4z^2}}$$

$$1) \frac{dz}{dt} = \sqrt{C_1 - \frac{1}{4z^2}}$$

$$\frac{2z dz}{\sqrt{4C_1 z^2 - 1}} = dt \Rightarrow \frac{1}{2C} \sqrt{4C_1 z^2 - 1} = t + C_2$$

$$\sqrt{4C_1 z^2 - 1} = 2C_2 t + C_3$$

$$4C_1 z^2 - 1 = (2C_2 t + C_3)^2, \quad z = \frac{y}{\sqrt{x}}, \quad t = \ln x$$

$$4C_1 \frac{y^2}{x} - 1 = (2C_2 \ln x + C_3)^2$$

✓ 457

$$yy'' = y'(y'+1), \quad yy' = y'y' + y'$$

$$yy'' - y'y' = y' \cdot 1 \cdot y^2$$

$$\frac{yy''}{y} - \left(\frac{y'}{y}\right)' = \frac{y'}{y^2}$$

$$\left(\frac{y'}{y}\right)' = \left(\frac{1}{y}\right)' \quad \frac{y'}{y} = -\frac{1}{y} + C_1$$

$$y' - C_1 y = -1$$

$$\frac{dy}{y} = \frac{C_1}{y} dx, \quad \ln|y| = C_1 x + C_2$$

$$y = e^{-x C_1 - C_2}$$

$$y = a, \quad a = -\frac{1}{C_1}, \quad a = -\frac{1}{C_1}, \quad y = C_2 e^{-C_1 x} - \frac{1}{C_1}$$

$$y = 0$$

✓ 459

$$yy'' + (y')^2 = 1$$

$$(yy')' = 1, \quad yy' = x + C_1$$

$$\int y dy = \int (x + C_1) dx, \quad \frac{y^2}{2} = \frac{x^2}{2} + x C_1 + C_2$$

✓ 501

$$\begin{cases} yy'' = 2x(y')^2 \\ y(2) = 0 \\ y'(2) = 0,5 \end{cases}$$

$$y'(x) = y(x) z(x)$$



$$(y = \text{const}) ; z^2 + z' = 2xz^2$$

$$\frac{z'}{z^2} = 2x - 1, \quad -\frac{1}{z} = x^2 - x + C_1$$

$$z = \frac{y'}{y}; \quad z(2) = \frac{1}{4};$$

$$-4 = 4 - 2 + C_1, \quad C_1 = -6;$$

$$-\frac{y'}{y^2} - \frac{y'}{y} = x^2 - x - 6,$$

$$\frac{y'}{y} = -\frac{1}{x^2 - x - 6} ; \quad \frac{dy}{y} = \frac{dx}{3} \left(\frac{1}{x+2} - \frac{1}{x-3} \right)$$

$$5 \ln y = \ln |x+2| - \ln |x-3| + C_2$$

$$y^5 = C_2 \frac{x+2}{x-3}, \quad z^5 = C_2 \frac{y}{1} \Rightarrow C_2 = 8$$

$$y^5 = 8 \frac{x+2}{x-3}$$

458

$$5(y^{(4)})^2 - 3y^{(4)}y^{(4)} = 0$$

$$z(x) = y^{(4)}, \quad 5(z')^2 - 3zz'' = 0$$

$$z' = u, \quad z'' = u' \cdot u,$$

$$5u^2 - 3zu'u = 0,$$

$$5u - 3zu' = 0,$$

$$5u = 3z \frac{du}{dz} \Rightarrow \int \frac{du}{u} = \frac{3}{5} \int \frac{dz}{z} = \ln |z| + C,$$

$$\frac{5}{2} \ln |z| + C_1 = \ln |u|,$$

$$u = C_1 z^{5/3}, \quad z' = C_1 z^{5/3}$$

$$\frac{dz}{dx} = C_1 z^{5/3}, \quad z^{-5/3} dz = C_1 dx$$

$$-\frac{3}{2} z^{-2/3} = C_1 x + C_2;$$

$$z^{-2/3} = C_1 x + C_2,$$

$$z = (C_1 x + C_2)^{-3/2}$$

$$y'' = (C_1 x + C_2)^{-3/2}$$

$$C_1 \neq 0; \quad y' = \int (C_1 x + C_2)^{-3/2} dx =$$

...

2/p

460

$$y'' = xy' + y + 1,$$

$$(y' - \frac{1}{y})' = (xy)'$$

$$y' - \frac{1}{y} = xy + C_1,$$

$$y' + y(-1-x) = C_1 + x$$

$$\frac{dy}{y} = (x+x) dx,$$

$$\ln |y| = x + \frac{x^2}{2} + C_2$$

$$y = C_2 e^{x + \frac{x^2}{2}}$$

$$y' = C_2 e^{x + \frac{x^2}{2}} (C_2' + (x+1)C_2)$$

$$e^{x + \frac{x^2}{2}} (C_2' + C_2(x+1)) + C_2 e^{x + \frac{x^2}{2}} (-1-x) = C_1 + x$$

$$C_2' = e^{-x - \frac{x^2}{2}} (C_1 + x)$$

$$C_2 = \int C_1 e^{-x - \frac{x^2}{2}} dx$$

$$C_2 = \int e^{-\frac{x^2}{2}} (C_1 + x) dx$$

$$= C_1 \int e^{-\frac{x^2}{2}} dx + \frac{1}{2} \int e^{-\frac{x^2}{2}} dx^2$$

$$= C_1 \int e^{-\frac{x^2}{2}} dx - e^{-\frac{x^2}{2}}$$

$$y = e^{\frac{x^2}{2}} (C_1 \int e^{-\frac{x^2}{2}} dx - e^{-\frac{x^2}{2}})$$

475

$$\frac{y}{x^2} + y'' = 3xy'' + \frac{2y}{x}$$

$$2m-2 = 2(m-1) = 1+m-2 = m+m-1-1 \Rightarrow m=1$$

$$x = e^t$$

$$y = ze^t$$

$$y' = (z' + z) \cdot e^t$$

$$y'' = (z'' + z') \cdot e^t$$

$$\frac{z e^{2t}}{e^t} + z' + 2z z' + z^2 =$$

$$= 3z(z' + z') + \frac{2z e^{2t} (z' + z)}{e^t}$$



$$z^2 + z'' + 2z' + z^2 = 3z'' + 3z' + 2z^2 + 2z^2$$

$$3z'' + 3z' + z^2 + (z')^2 = 0$$

$$\begin{cases} z = \cos t \\ y = \cos^2 t \end{cases}$$

$$z' = p(t) \quad 3p' + 3p - p^2 = 0 \quad | \cdot p'$$

$$3 \frac{p'}{p^2} + \frac{3}{p} - 1 = 0, \quad u = \frac{1}{p}, \quad u' = -\frac{1}{p^2} p'$$

$$-3u' + 3u - 1 = 0, \quad u' - u = -\frac{1}{3}$$

$$\frac{du}{u - \frac{1}{3}} = dt$$

$$\ln |u - \frac{1}{3}| = t + C_1$$

$$u - \frac{1}{3} = C_1 e^t$$

$$\frac{dt}{dz} = \frac{1}{p} = u = C_1 e^t + \frac{1}{3}$$

$$\Rightarrow z' = \frac{1}{C_1 e^t + \frac{1}{3}} \Rightarrow z = \int \frac{d(e^t)}{e^t(C_1 e^t + \frac{1}{3})} \quad \textcircled{c}$$

$$1) p_1 = \infty \Rightarrow z = C_2 \quad \textcircled{c} \quad \int \frac{d(e^t)}{e^t} = \int \frac{d(e^t)}{C_1 e^t + \frac{1}{3}}$$

$$\textcircled{c} \quad 3 \ln |e^{3t}| - \ln |C_1 e^t + \frac{1}{3}|^3 + C_2$$

$$t = \ln x, \quad z = \frac{y}{x} \Rightarrow \frac{y}{x} = \ln \frac{x^3}{C_1 x + \frac{1}{3}} + C_2$$

$$y = x \ln \frac{x^3}{C_1 x + \frac{1}{3}} + C_2, \quad C_2 \neq \infty$$

503

$$x^2 y'' - 3xy' = \frac{6y}{x^2} - 4y, \quad y(1) = 1, \quad y'(1) = 2$$

$$2 + m - 2 = 1 + m - 1 = 2m - 2 \quad | \quad y' = y^m(x)$$

$$\begin{cases} x = e^t \\ y = e^{2t} u(t) \end{cases}, \quad u'' - 6u' = 0, \quad | \cdot u' \Rightarrow u'^2 = 4u^3 + C_1(u)$$

$$T.K. \quad y(1) = 1 \Rightarrow u(0) = 1, \quad T.K.$$

$$y = e^t(u + 2u) \quad u \quad y'(1) = 4 \Rightarrow u'(0) + 2u(0) = 4$$

$$\Rightarrow u'(0) = 2. \quad \text{Положим } b(1) \quad t=0$$

$$u, \text{ переменная } \textcircled{c} \text{ переменная } u(0), u'(0) \Rightarrow$$

$$\Rightarrow C_1 = 0, \quad u'^2 = 4u^3 \Rightarrow u' = \pm 2u^{\frac{3}{2}} \quad \pm u^{-\frac{3}{2}} du = 2dt$$

$$z = \frac{1}{\pm \sqrt{u}} = t + C_2 \quad \text{или} \quad u = \frac{1}{(t + C_2)^2}$$

$$u(0) = 1 \Rightarrow C_2 = \pm 1, \quad u = \frac{1}{(t \pm 1)^2} \quad u(0) = 2 \Rightarrow$$

$$\Rightarrow u = \frac{1}{(t+1)^2}, \quad T.K. \quad y = \frac{x^2}{(\ln x - 1)^2}$$

Кн. работа

511

$$y'' + y' - 2y = 0, \quad \lambda^2 + \lambda - 2 = 0$$

$$\lambda = \frac{-1 \pm \sqrt{1+8}}{2} = \begin{Bmatrix} 1 \\ -2 \end{Bmatrix}$$

$$y(x) = C_1 e^x + C_2 e^{-2x}$$

513

$$y'' - 2y' = 0, \quad \lambda^2 - 2\lambda = 0, \quad \lambda = \begin{Bmatrix} 0 \\ 2 \end{Bmatrix}$$

$$y(x) = C_1 + C_2 e^{2x}$$

529

$$y^{(4)} - 5y'' + 4y = 0$$

$$\lambda^4 - 5\lambda^2 + 4 = 0, \quad \lambda = \begin{Bmatrix} 0 \\ 1 \\ -1 \end{Bmatrix}$$

$$\Delta(\lambda) = 5\lambda^2 + 4 = 0$$

$$\Delta = 25 - 16 = 9$$

$$y(x) = C_1 e^{2x} + C_2 e^{-2x} + C_3 e^x + C_4 e^{-x}$$

515

$$y'' - 4y' + 5y = 0$$

$$\lambda^2 - 4\lambda + 5 = 0, \quad \lambda = 2 \pm i$$

$$y(x) = C_1 e^{(2+i)x} + C_2 e^{(2-i)x}, \quad e^{ix} = \cos x + i \sin x$$

$$e^{2x} \cos x, e^{2x} \sin x \quad | \quad y = e^{2x} (C_1 \cos x + C_2 \sin x)$$

$$e^{(2+i)x} \quad e^{(2-i)x}$$

516

$$y'' + 2y' + 10y = 0$$

$$\lambda^2 + 2\lambda + 10 = 0, \quad \Delta = 4 - 40 = -36$$

$$\lambda_{1,2} = \frac{-2 \pm 6i}{2} = -1 \pm 3i$$

$$y(x) = e^{-x} (C_1 \cos 3x + C_2 \sin 3x)$$

519

$$y^{(4)} - y = 0$$

$$\lambda^4 - 1 = 0, \quad \lambda^2 = 1, \quad \lambda = \pm 1$$

$$\lambda = \pm i$$

$$y = C_1 e^x + C_2 e^{-x} + C_3 \sin x + C_4 \cos x$$

5521

$$4y'' + 4y' + y = 0,$$

$$4\lambda^2 + 4\lambda + 1 = 0, \Delta = 0$$

$$(2\lambda + 1)^2 = 0, \lambda = -\frac{1}{2}$$

$$y = (c_1 + c_2 x) e^{-\frac{x}{2}}$$

5522

$$y''' - 3y'' + 3y' - y = 0$$

$$\lambda^3 - 3\lambda^2 + 3\lambda - 1 = 0$$

$$(\lambda - 1)^3 = 0, y = e^x (c_1 + c_2 x + c_3 x^2)$$

5530

$$y'' + 8y''' + 16y' = 0$$

$$\lambda^5 + 8\lambda^3 + 16\lambda = 0,$$

$$\lambda(\lambda^4 + 8\lambda^2 + 16) = 0,$$

$$(\lambda^2 + 4)^2, \lambda = 0$$

$$\lambda^2 = -4$$

$$\lambda = \pm 2i$$

$$y(x) = c_1 + (c_2 + c_3 x) e^{2ix} + (c_4 + c_5 x) e^{-2ix}$$

5585

$$x^2 y'' - 4x y' + 6y = 0$$

$$x = e^t, y' = \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} =$$

$$= \dot{y} e^{-t}, y'' = \frac{d^2 y}{dx^2} = \frac{d\dot{y}}{dt} \cdot \frac{dt}{dx} =$$

$$= (\dot{y} e^{-t})' e^{-t} = \ddot{y} e^{-t} + \dot{y} e^{-t} e^{-t} =$$

$$= \ddot{y} e^{-2t} + \dot{y} e^{-2t} \Rightarrow e^{2t} \cdot e^{-2t} (\ddot{y} + \dot{y})$$

$$- 4e^t \cdot e^{-t} \dot{y} + 6y = 0,$$

$$\ddot{y} + \dot{y} - 4\dot{y} + 6y = 0,$$

$$\ddot{y} - 3\dot{y} + 6y = 0$$

$$\lambda^2 - 3\lambda + 6 = 0, \Delta = 25 - 4 \cdot 6 = 1$$

$$\lambda = \frac{3 \pm 1}{2} \quad y(x) = c_1 e^{4x} + c_2 e^{5x},$$

$$y(x) = c_1 x^2 + c_2 x^3$$

5592

$$x^2 y''' = 2y' \cdot x, \quad x^3 y''' - 2xy' = 0$$

$$y''' = (e^{-2t} (\ddot{y} - \dot{y}))' e^{-t} =$$

$$= e^{-3t} (-2\ddot{y} + 2\dot{y} + \ddot{y} - \dot{y}) = e^{-3t} (\ddot{y} - \dot{y} + 2\dot{y})$$

$$e^{3t} + e^{-3t} (\ddot{y} - \dot{y} + 2\dot{y}) - 2\dot{y} e^{-t} \cdot e^t = 0$$

$$\ddot{y} - 3\ddot{y} + 2\dot{y} - 2\dot{y} = 0,$$

$$\ddot{y} - 3\dot{y} = 0, \lambda^2 - 3\lambda = 0, \lambda = 0$$

$$\lambda = 3$$

$$y(t) = c_1 e^{3t} + c_2 + c_3 t$$

$$y(x) = c_1 x^3 + c_2 + c_3 \ln|x|$$

615

$$y_1 = x \sin x, \quad \lambda = i \text{ krasn. 2}$$

$$(\lambda - i)^2 = 0, \quad \lambda^2 + 1 = 0$$

$$(\lambda + i)^2 = 0$$

$$x \sin x, \lambda = \pm i \quad (\lambda^2 + 1)^2 = 0$$

$$\sin x \text{ krasn. 2} \quad \lambda^4 + 2\lambda^2 + 1 = 0$$

$$y^{(4)}(x) + 2y''(x) + y(x) = 0.$$

617

$$y = x e^x, y_2 = e^{-x}$$

$$\lambda = 1 \quad \lambda = -1$$

$$\lambda^2 - 1 = 0, \quad (\lambda - 1)^2 (\lambda + 1) = 0$$

$$(\lambda^2 - 2\lambda + 1)(\lambda + 1) = 0$$

$$\lambda^3 - \lambda^2 - \lambda + 1 = 0$$

$$y''' - y'' - y' + y = 0$$

5533

$$y'' - 2y' - 3y = e^{4x}$$

$$y'' - 2y' - 3y = 0,$$

$$\lambda^2 - 2\lambda - 3 = 0, \lambda = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$$y_0 = c_1 e^{3x} + c_2 e^{-x}$$

$$y_2 = a e^{4x}, \quad y_2'' = 16a e^{4x}$$

$$y_1' = 4a e^{4x}$$

$$16e^{4x} - 8ae^{4x} - 4e^{4x} = e^{4x}$$

$$a = \frac{1}{5} \Rightarrow y = y_0 + y_2 =$$

$$= c_1 e^{5x} + c_2 e^{-x} + \frac{1}{5} e^{4x}$$

5513

$$y'' - 5y' + 4y = 0 \quad \lambda^2 - 5\lambda + 4 = 0, \Delta = 25 - 16 = 9$$

$$\lambda_{1,2} = \frac{5 \pm 3}{2} = 4; 1$$

$$y_0 = c_1 e^{4x} + c_2 e^x$$

$$y_2 = (ax^2 + bx + c)e^{2x},$$

$$y_2' = (2ax + b)e^{2x} + 2(ax^2 + bx + c)e^{2x}$$

$$= e^{2x}(2ax^2 + 2(2a + b)x + b + 2c)$$

$$y_2'' = e^{2x}(4ax + 2(4a + b)) + 2(2ax + b)e^{2x}$$

$$+ e^{2x}(4ax + 2a + 2b)$$

$$4ax^2 + 4ax + 4bx + 2bx + 4c + 4ax + 2a + 2b$$

$$- 10ax^2 - 10ax - 10bx - 10(-5b +$$

$$+ 4ax^2 + 4bx + 4c \neq 4x^2$$

$$-2a = 4, \quad a = -2$$

$$-2a = 2b; \quad b = -2$$

$$-2a - b - 2c + 2a = 0, \quad c = -3$$

$$y = y_0 + y_2 = c_1 e^{4x} + c_2 e^x + (-2x^2 + 2x - 3)e^{2x}$$

27p

5519

$$y^{(4)} - y = 0, \quad \lambda^4 - 1 = 0$$

$$\lambda^4 = 1, \quad \lambda^2 = \pm 1$$

$$\lambda = \begin{bmatrix} \pm 1 \\ \pm i \end{bmatrix}$$

$$y = c_1 e^{-x} + c_2 e^x + c_3 \sin x + c_4 \cos x$$

5514

$$2y'' - 5y' + 2y = 0$$

$$2\lambda^2 - 5\lambda + 2 = 0, \Delta = 25 - 16 = 9$$

$$\lambda = \frac{5 \pm 3}{4} = 2; \frac{1}{2}$$

$$y = c_1 e^{2x} + c_2 e^{\frac{1}{2}x}$$

5525

$$y^{(5)} - 10y''' + 9y' = 0$$

$$\lambda^5 - 10\lambda^3 + 9\lambda = 0, \quad \lambda(\lambda^4 - 10\lambda^2 + 9) = 0$$

$$y = c_1 + c_2 e^{2x} + c_3 e^{-2x} + c_4 e^{3x} + c_5 e^{-3x}$$

5517

$$y'' + 4y = 0, \quad \lambda^2 + 4 = 0, \quad \lambda = \pm 2i, \quad y = c_1 \sin 2x + c_2 \cos 2x$$

5518

$$y''' - 8y = 0, \quad \lambda^3 - 8 = 0, \quad \lambda = 2, \quad y_1 = c_1 e^{2x}$$

$$(x-2)(\lambda^2 + b\lambda + c) = 0$$

$$\lambda^3 + b\lambda^2 + c\lambda - 2 = 0$$

$$b = -2, \quad c = 2$$

$$(\lambda - 2)(\lambda^2 + 2\lambda + 4) = 0$$

$$\Delta = 4 - 16 = -12$$

$$\lambda_{1,2} = \frac{-2 \pm \sqrt{-12}}{2} = -1 \pm \sqrt{3}i$$

$$y = c_1 e^{2x} + c_2 e^{-1 + \sqrt{3}i} x + c_3 e^{-1 - \sqrt{3}i} x$$

5520

$$y^{(4)} + 4y = 0, \quad \lambda^4 + 4 = 0, \quad \lambda^2 = \pm 2i, \quad \lambda = \begin{bmatrix} \pm \sqrt{2}i \\ \pm i\sqrt{2}i \end{bmatrix}$$

$$\lambda_k = \sqrt[4]{-4} = \sqrt{2} e^{\frac{i(\pi + 2\pi k)}{4}}$$

$$(i)^{\frac{1}{2}} = \left(\cos\left(\frac{\pi}{2} + 2\pi k\right) + i \sin\left(\frac{\pi}{2} + 2\pi k\right) \right)^{\frac{1}{2}}$$

$$= \left(\cos \frac{\pi + 4\pi k}{2} + i \sin \frac{\pi + 4\pi k}{2} \right), \quad k = 0, 1$$

$$= \begin{bmatrix} \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \\ \cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} \end{bmatrix} = \begin{bmatrix} i \\ -i \end{bmatrix}$$

$$\lambda_1 = 1 + i, \quad \lambda_2 = 1 - i, \quad \lambda_3 = -1 + i, \quad \lambda_4 = -1 - i$$

$$y = c_1 \cos x + c_2 \sin x + c_3 e^{(1+i)x} + c_4 e^{(1-i)x}$$

524

$$y^{IV} - 6y''' + 9y'' = 0$$

$$\lambda^4 - 6\lambda^3 + 9\lambda^2 = 0, \quad \lambda^2(\lambda^2 - 6\lambda + 9) = 0$$

$$\lambda = 0$$

$$(\lambda - 3)^2$$

$$y = (C_1 + C_2x + C_3x^2) + (C_4 + C_5x)e^{3x}$$

526

$$y^{IV} + 2y'' + y = 0$$

$$\lambda^4 + 2\lambda^2 + 1 = 0, \quad (\lambda^2 + 1)^2 = 0$$

$$\lambda^2 = -1, \quad \lambda = \pm i$$

$$y = C_1 \sin x + C_2 \cos x + C_3 x \sin x + C_4 x \cos x$$

528

$$y''' + 3y'' + 3y' + y = 0$$

$$\lambda^3 + 3\lambda^2 + 3\lambda + 1 = 0$$

$$\lambda^2(\lambda + 1) + (\lambda + 1) = 0$$

$$(\lambda + 1)(\lambda^2 + 1) = 0, \quad (\lambda + 1)^2(\lambda + 1) = 0$$

$$y = (C_1 + C_2x)e^{-x} + C_3e^{-x}$$

531

$$x^3y''' + xy' - y = 0$$

$$x = e^t, \quad y' = \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = y' e^{-t}$$

$$y'' = \frac{d^2y}{dx^2} = \frac{d^2y}{dt^2} \cdot \frac{dt^2}{dx^2} = y'' e^{-2t}$$

$$y''' = e^{-3t}(-2y'' + 2y' + y'' - y') = e^{-3t}(y'' - 3y' + 2y')$$

$$= e^{-3t}(y'' - 3y' + 2y')$$

$$e^{3t}e^{-3t}(y'' - 3y' + 2y') + y' - y = 0$$

$$y'' - 3y' + 3y' - y = 0$$

$$\lambda^3 - 3\lambda^2 + 3\lambda - 1 = 0$$

$$(\lambda - 1)(\lambda^2 + \lambda + 1) + 3\lambda(\lambda - 1) = 0$$

$$(\lambda - 1)(\lambda^2 - 2\lambda + 1) = 0, \quad (\lambda - 1)(\lambda - 1)^2 = 0$$

$$y = (C_1 + C_2x + C_3x^2)e^{3t}$$

$$u = (C_1 + C_2x + C_3x^2)x^3$$

500

$$(2x+3)^2y''' + 3(2x+3)y' - 6y = 0$$

$$2x+3 = u, \quad u^2y''' + 3uy' - 6y = 0$$

$$u = e^t, \quad y' = \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = y' e^{-t}$$

$$y''' = 8e^{-3t}(y'' - 3y' + 2y')$$

$$8y'' - 24y' + 16y' + 6y' - 6y = 0$$

$$8y'' - 24y' + 22y' - 6y = 0$$

$$4y'' - 12y' + 11y' - 3y = 0$$

$$4\lambda^2 - 12\lambda + 11\lambda - 3 = 0$$

$$(\lambda - 1)(4\lambda^2 - 8\lambda + 3) = 0$$

$$y = C_1(x + \frac{3}{2}) + C_2(x + \frac{3}{2})^{\frac{1}{2}} + C_3(x + \frac{3}{2})^{\frac{3}{2}}$$

566

$$y_1 = x e^x \cos 2x, \quad y_2 = x e^x \sin 2x$$

$$\Rightarrow \lambda_{1,2} = 1 \pm 2i \quad \text{or } 2 \quad (\lambda - 1 + 2i)^2 (\lambda - 1 - 2i)^2 = 0$$

$$(\lambda - 1 + 2i)^2 (\lambda - 1 - 2i)^2 = 0$$

$$(\lambda^2 - 2\lambda + 1 + 4)^2 = 0, \quad (\lambda^2 - 2\lambda + 5)^2 = 0$$

$$\lambda^4 + 4\lambda^2 + 25 - 4\lambda^3 + 10\lambda^2 - 20\lambda = 0$$

$$\lambda^4 - 4\lambda^3 + 14\lambda^2 - 20\lambda + 25 = 0$$

$$y'' - 4y' + 14y'' - 20y' + 25y = 0$$

568

$$y_1 = x, \quad y_2 = \sin x, \quad \lambda_1 = 0 \quad \text{or } 2$$

$$\lambda_2 = \pm i \quad \text{or } 1$$

$$\lambda^3 + 2\lambda^2 + \lambda = 0$$

$$y'' + 2y' + y = 0$$

534

$$y'' + y = 4xe^x, \quad \lambda^2 + 1 = 0, \quad \lambda = \pm i$$

$$y_2 = (ax + b)e^x$$

$$y_2' = e^x(a + ax + b) = e^x(ax + b + a)$$

$$y_2'' = e^x(ax + b + 2a), \quad ax + b + 2a + ax + b = 4x$$

$$2ax + 2(b + a) = 4x$$

$$y = C_1 \cos x + C_2 \sin x + (2x-2)e^x$$

5542

$$y'' + 4y' + 4y = xe^{2x}, \quad \lambda^2 + 4\lambda + 4 = 0$$

$$(\lambda + 2)^2 = 0$$

$$\lambda = -2$$

$$y_h = (C_1 + C_2 x)e^{-2x}$$

$$y_2 = (x + b)e^{2x}$$

$$y_2' = e^{2x}(a + 2ax + b) = e^{2x}(2ax + 2b + a)$$

$$y_2'' = e^{2x}(2a + 4ax + 4b + 2a) = e^{2x}(4ax + 4b + 4a)$$

$$4ax + 4b + 4a + 8ax + 8b + 4a +$$

$$+ 4ax + 4b = x, \quad x(4a + 8a + 4a) + 4b + 4a + 8b + 4a$$

$$+ 4b = x,$$

$$16a = 1, \quad 16b + 8a = 0$$

$$a = \frac{1}{16}, \quad 16b + \frac{1}{2} = 0$$

$$32b + 1 = 0$$

$$b = -\frac{1}{32}$$

$$y = \frac{1}{16}\left(x - \frac{1}{2}\right)e^{2x} + (C_1 + C_2 x)e^{-2x}$$

Kn. präs. 54

5545

$$y'' - 2y' + y = 6xe^x$$

$$1) y'' - 2y' + y = 0, \quad \lambda^2 - 2\lambda + 1 = 0,$$

$$(\lambda - 1)^2 = 0, \quad \lambda = 1 \text{ m. l.}$$

$$y_h = (C_1 + C_2 x)e^x$$

$$2) y_2 = (ax^2 + bx)e^x$$

$$y_2 = (ax^2 + bx)e^x, \quad y_2' = e^x(ax^2 + bx^2 + 2ax + 3bx^2 +$$

$$y_2'' = e^x(ax^2 + bx^2 + 2ax + 3bx^2 + 2ax + 3bx^2 +$$

$$+ 2a + 6bx),$$

$$(ax^2 + bx^2 + 4ax + 6bx^2 + 2a + 6bx)$$

$$- 2(ax^2 + bx^2 + 2ax + 3bx^2) + (ax^2 + bx^2) = 6x$$

$$\begin{cases} 6b = 6 & b = 1 \\ 2a = 0 & a = 0 \end{cases} \Rightarrow y_2 = x^3 e^x$$

$$\Rightarrow y = y_2 + y_h = x^3 e^x + (C_1 + C_2 x)e^x$$

5538

$$y'' + y = 4 \sin x - 4e^{ix}$$

$$1) y'' + y = 0, \quad \lambda^2 + 1 = 0, \quad \lambda = \pm i$$

$$y_h = C_1 \cos x + C_2 \sin x$$

$$2) y_2 = a \cdot x \cdot e^{ix}$$

$$y_2' = ae^{ix} + iaxe^{ix}, \quad y_2'' = 2iae^{ix} - axe^{ix}$$

$$\text{Charakteristik} \Rightarrow a = \frac{2}{i} = -2i$$

$$y_2 = -2ix(\cos x + i \sin x)$$

$$\operatorname{Im}(y_2) = -2x \cos x$$

$$y_0 = y_h + y_2 = C_1 \cos x + C_2 \sin x - 2x \cos x$$

5535

$$y'' - y = 2e^x - x^2$$

$$1) y'' - y = 0, \quad \lambda^2 - 1 = 0, \quad \lambda = \pm 1$$

$$y_h = C_1 e^x + C_2 e^{-x}$$

$$2) y_2 = ax^2 e^x$$

$$y_2 = ax^2 e^x, \quad y_2' = e^x(a + 2ax) = ae^x(1 + 2x)$$

$$y_2'' = ae^x(1 + 2x) + ae^x = ae^x(2 + 2x)$$

$$\text{Integration} \Rightarrow y_{21} = xe^x$$

$$2) y'' - y = -x^2$$

$$y_2 = ax^2 + bx + c, \quad y_2' = 2ax + b,$$

$$y_2'' = 2a, \quad \begin{cases} a = 1 \\ b = 0 \\ c = 2 \end{cases}$$

$$y_{22} = x^2 + 2$$

$$y = y_h + y_{21} + y_{22} = C_1 e^x + C_2 e^{-x} + xe^x + x^2 + 2$$

5541

$$y'' + 3y' - 4y = e^{-4x} + xe^{-x}$$

$$1) y'' + 3y' - 4y = 0, \quad \lambda = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$$

$$y_h = C_1 e^x + C_2 e^{-4x}$$

$$2) y'' + 3y' - 4y = e^{-4x}$$

$$y_2 = ax^2 e^{-4x}, \quad y_2' = ae^{-4x} - 4ax e^{-4x}$$

$$y_2'' = -4ae^{-4x} - 4ae^{-4x} + 16axe^{-4x}$$

$$-8a + 16ax = 1$$

$$a = -\frac{1}{8} \Rightarrow y_{21} = -\frac{x}{8} e^{-4x}$$

$$2) y'' + 3y' - 4y = xe^{-x}$$

$$y_2 = (ax + b)e^{-x} = axe^{-x} + be^{-x}$$

$$y_2' = ae^{-x} - axe^{-x} - be^{-x}$$

$$y_2'' = -ae^{-x} - ae^{-x} + axe^{-x} + be^{-x}$$

logarithmieren $\Rightarrow -2a+ax+b+3a-3ax-3b$
 $-6a-4b=x \Rightarrow \begin{cases} -6a=1 & a=-\frac{1}{6} \\ a-6b=0 & b=-\frac{1}{36} \end{cases}$

$$y_2 = \left(\frac{1}{6}x - \frac{1}{36}\right)e^{-x}$$

$$y = y_0 + y_1 + y_2$$

5555

$$y'' + 2y' + 2y = xe^{-x}; \quad y(0)=0; y'(0)=0$$

1) $\lambda = -1 \pm i, y_0 = e^{-x}(C_1 \sin x + C_2 \cos x)$
 nur periodisch $\Rightarrow$

$\Rightarrow 2) y_2 = (ax+b)e^{-x}, y_2' = e^{-x}(a-b-ax)$
 $y_2'' = e^{-x}(-a-a+ax+b)$

$$\begin{cases} a=b \\ b=0 \end{cases} \quad y_2 = xe^{-x}$$

$$\Rightarrow y = e^{-x}(C_1 \sin x + C_2 \cos x + x)$$

$$y(0) = C_2 = 0; \quad y'(x) = e^{-x}(C_1 \sin x + x + 1 + C_2 \cos x)$$

$$y'(0) = 1 + C_1 = 0 \Rightarrow C_1 = -1$$

5595

$$x^2 y'' - xy' + y = 8x^3$$

$$x = e^t, y' = e^{-t} \dot{y}, y'' = e^{-t}(-\dot{y} + \ddot{y}) \cdot e^{-t} =$$

$$= e^{-2t}(\ddot{y} - \dot{y}); \quad 1) \ddot{y} - 2\dot{y} + 1 = 8e^{3t}$$

$$\Rightarrow y(t) = (C_1 + tC_2)e^t \quad (\text{wegen } \cos \sin)$$

$$y_2 = ae^{3t}, y_2' = 3ae^{3t}, y_2'' = 9ae^{3t}$$

$$a=2, y(t) = (C_1 + C_2 t)e^t + 2e^{3t}$$

$$y(x) = (C_1 + C_2 \ln(x))x + 2x^3$$

5576

$$y'' + 3y' + 2y = \frac{1}{e^{x+1}}$$

1) $\lambda^2 + 3\lambda + 2 = 0, \Delta = 9 - 8 = 1, \lambda = \frac{-3 \pm 1}{2} =$

$$y_0(x) = C_1 e^{-x} + C_2 e^{-2x}$$

$$= \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

$$\begin{cases} C_1 e^{-x} + C_2 e^{-2x} = 0 \\ -C_1 e^{-x} - 2C_2 e^{-2x} = \frac{1}{e^{x+1}} \end{cases}$$

$$-C_1 e^{-x} - 2C_2 e^{-2x} = \frac{1}{e^{x+1}}$$

$$\begin{cases} \dot{C}_2 = -\frac{2e^x}{e^{x+1}} \Rightarrow C_2 = -\frac{2e^x}{e^{x+1}} \\ C_1' = -C_2' e^{-x} \end{cases}$$

$$\Rightarrow C_1 = \int \frac{2e^x}{e^{x+1}} dx = \int \frac{2}{e^{x+1}} dx =$$

$$= -\int \frac{2}{t+1} dt = -2 \int \frac{t+1-1}{t+1} dt =$$

$$= -2t + 2 \ln|t+1| + \tilde{C}_2 = -e^x + 2 \ln(e^{x+1}) + \tilde{C}_2$$

$$C_1 = \int \frac{e^x}{e^{x+1}} dx = \int \frac{1}{t+1} dt = \ln|t+1| + \tilde{C}_1$$

$$y(x) = (\ln(e^{x+1}) + \tilde{C}_1)e^{-x} + (-e^x + 2 \ln(e^{x+1}) + \tilde{C}_2)e^{-x}$$

$$y'' + by' + cy = f(x)$$

$$y_1, y_2 \text{ - l. u. l. per.}$$

$$y = C_1 y_1 + C_2 y_2, \quad y' = C_1' y_1 + C_2' y_2 + C_1 y_1' + C_2 y_2'$$

$$y'' = C_1' y_1' + C_2' y_2' + C_1 y_1'' + C_2 y_2''$$

5577

5578

$$y'' + 4y = 2 \ln x$$

$$\lambda^2 + 4 = 0, \quad y = C_1 \cos 2x + C_2 \sin 2x$$

$$\begin{cases} C_1' \cos 2x + C_2' \sin 2x = 0 \\ -2C_1' \sin 2x + 2C_2' \cos 2x = 2 \ln x \end{cases} \quad \begin{array}{c|c} \sin 2x & \cos 2x \\ \hline \cos 2x & -\sin 2x \end{array}$$

$$C_2' = \ln x \cos 2x$$

$$C_1' = -\sin 2x \ln x = -\frac{1}{2} \ln^2 x = \ln 2x - 1$$

$$C_1 = \frac{\sin 4x}{2} - x + \tilde{C}_1, \quad C_2 = \int \ln x \cos 2x dx =$$

$$= \int \frac{t \cos 2x}{dt} dt = -\int \frac{dt (2 \ln 2x)}{t} =$$

$$= -\ln^2 t + \ln t + \tilde{C}_2 = -\ln^2 x + \ln x + \tilde{C}_2$$

5540

$$y'' - 3y' + 2y = x \cos x = \operatorname{Re}(x e^{ix})$$

1) $y'' - 3y' + 2y = 0, \quad \lambda^2 - 3\lambda + 2 = 0,$

$$y_0 = C_1 e^{2x} + C_2 e^x$$

$$y_2 = C_1 x + C_2 x^2$$

$$\Delta = 9 - 4 = 5, \quad \lambda_{1,2} = \frac{3 \pm \sqrt{5}}{2} =$$

$$y_2 = (ax+b)e^{ix}, y_2' = ae^{ix} + (ax+b)e^{ix}i = e^{ix}(a + i(ax+b))$$

$$y_2'' = e^{ix}(a + i(a + i(ax+b))) = e^{ix}(a + i(2a + i(ax+b)))$$

$$a + i(2a + i(ax+b)) - 3a - 3i(ax+b) + 2ax + 2b = x$$

$$\begin{cases} a - 3a + 2a = 1 \\ b + 2ai - 3ai - 3b + 2b = 0 \end{cases} \Rightarrow \begin{cases} a = \frac{1}{3-i} \\ b + 2ai - 3ai - 3b + 2b = 0 \end{cases}$$

$$a = \frac{1}{3} \cdot \frac{1}{1-i} = \frac{1}{3} \cdot \frac{1+i}{2} = \frac{1}{6}(1+i)$$

$$b(1-i) + a(2i-3) = 0$$

$$b = \frac{a(2i-3)}{3-i} = \frac{1}{6} a(2i-3)(1+i) =$$

$$= \frac{1}{6} \cdot \frac{1}{6} (2i-3)(1+i) = \frac{1}{36} (2i-3)(1+i) =$$

$$= \frac{1}{18} (2i-3) = \frac{1}{18} (-2-3i) \Rightarrow$$

$$\Rightarrow y_2 = \left(\frac{1}{6}(1+i)x + \frac{1}{18}(-2-3i) \right) e^{ix} =$$

$$\Rightarrow \text{Re}[y_2] = \frac{x}{6} \cos x - \frac{1}{9} \cos x - \frac{x}{6} \sin x + \frac{1}{9} \sin x \Rightarrow y(x) = c_1 e^{ix} + c_2 e^{-ix} + \text{Re}[y_2]$$

544

$$y'' - 9y = e^{3ix} \cos x = \text{Re} [e^{(3+i)x}]$$

$$1) \lambda^2 - 9 = 0, \lambda = \pm 3$$

$$y_0 = c_1 e^{3x} + c_2 e^{-3x}$$

$$2) y_2 = a \cdot e^{(3+i)x}, y_2' = a e^{(3+i)x} (3+i)$$

$$y_2'' = a(3+i)^2 e^{(3+i)x} = a(8+6i) e^{(3+i)x}$$

$$\Rightarrow a(8+6i) - 9a e^{(3+i)x} = 1$$

$$a = \frac{1}{8+6i-9} = \frac{1}{-1+6i} =$$

$$= \frac{1}{1-i} \cdot (-1-6i) \Rightarrow y_2 = \frac{1}{1-i} (-1-6i) e^{(3+i)x} =$$

$$(1-i)(-1-6i) = (-1-6i) + i(1+6) = (-1-6i) + i(7) = (-1-6i) + i(7)$$

$$\Rightarrow y_2 = \frac{1}{1-i} (-1-6i) e^{(3+i)x} = \frac{1}{1-i} (-1-6i) e^{3x} (\cos x + i \sin x)$$

$$\text{Re}[y_2] = \frac{e^{3x}}{1-i} \left(\frac{-1-6i}{2} (\cos x + i \sin x) + \frac{1-6i}{2} (\sin x - i \cos x) \right)$$

$$= e^{3x} \left(-\frac{1}{2} \cos x + \frac{1}{2} \sin x \right)$$

$$y = y_0 + y_2 = c_1 e^{3x} + c_2 e^{-3x} + e^{3x} \left(-\frac{1}{2} \cos x + \frac{1}{2} \sin x \right)$$

546

$$y'' + y' - 2y = 3xe^x$$

$$1) \lambda^2 + \lambda - 2 = 0, \Delta = 1+8=9$$

$$\lambda_{1,2} = \frac{-1 \pm 3}{2} = \begin{cases} 1 \\ -2 \end{cases}$$

$$\Rightarrow y_0 = c_1 e^x + c_2 e^{-2x}$$

$$2) y_2 = a x e^x (ax+b) e^{-x}$$

$$y_2' = e^x (a - ax - b) = e^x (a - b - ax)$$

$$y_2'' = e^x (a - b - ax) = e^x (a - b - ax)$$

$$-2a + b + ax + a - b - ax - 2ax - 2b = 3x$$

$$\begin{cases} -3a - 2b = 0 \\ -2a = 3 \end{cases} \Rightarrow \begin{cases} b = -\frac{3}{2}a \\ a = -\frac{3}{2} \end{cases}$$

$$y_2 = \left(-\frac{3}{2}x + \frac{9}{4} \right) e^{-x} \quad y = y_0 + y_2$$

546

$$y'' + y = x \cos x = \text{Re} [\text{Im}(x e^{ix})]$$

$$1) \lambda^2 + 1 = 0, \lambda = \pm i; y_0 = c_1 \cos x + c_2 \sin x$$

$$2) y_2 = a x e^{ix} (ax+b) e^{-ix}$$

$$y_2' = e^{ix} (a + i(ax+b)), y_2'' = e^{ix} (a + i(2a + i(ax+b)))$$

$$-ax + b + 2ai + a + i(2a + i(ax+b)) = x$$

$$-a + b = 1$$

$$-b + 2ai + a = 0 \Rightarrow a = \frac{-1}{2i} = +\frac{1}{2} \cdot i = \frac{i}{2}$$

$$b = 1 + \frac{i}{2} = \frac{2+i}{2}$$

$$\Rightarrow y_2 = \left(\frac{i}{2}x + 1 + \frac{i}{2} \right) e^{ix} (\cos x + i \sin x)$$

$$\text{Im}(y_2) = \frac{1}{2} x \cos x \sin x \left(\frac{x}{2} \right) + \cos x \left(\frac{x}{2} + \frac{1}{2} \right)$$

$$y_2 = (ax+bx^2) e^{ix}, y_2' = e^{ix} (a + 2bx + ax + iax^2)$$

$$y_2'' = e^{ix} (2b + a + 2bix + ai + 2bix - ax - bx^2)$$

$$i(2a+4bx) + 2b - bx^2 + bx + bx^2 = x$$

$$\begin{cases} 4bi = 1, & b = \frac{1}{4} \cdot \frac{1}{i} = -\frac{1}{4}i \\ 2ai + 2b = 0, & a = -\frac{b}{i} = +\frac{1}{4} \end{cases}$$

$$\Rightarrow y_2 = (\frac{1}{4}x + \frac{1}{4}ix^2)(\cos x + i\sin x)$$

$$\Rightarrow \operatorname{Im}(y_2) = \cos x(-\frac{1}{4}x^2) + \sin x(\frac{1}{4}x)$$

$$\Rightarrow y = (C_1 + \frac{1}{4}x)\sin x + (C_2 - \frac{1}{4}x^2)\cos x$$

548

$$y'' - 5y' = 3x^2 + 8\sin x$$

$$1) y'' - 5y' = 0$$

$$\lambda^2 - 5\lambda = 0, \lambda(\lambda - 5) = 0 \Rightarrow \lambda = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

$$1) y'' - 5y' = 3x^2 \quad y_0 = C_1 + C_2 e^{5x}$$

$$\Rightarrow y_2 = ax^3 + bx^2 + cx + d$$

$$y_2' = 3ax^2 + 2bx + c, y_2'' = 6ax + 2b$$

$$\Rightarrow 6ax + 2b - 15ax^2 - 10bx - 5c = 3x^2$$

$$\begin{cases} 6a - 10b = 0, & +6 \cdot \frac{1}{5} + 10b = 0, & b = -\frac{6}{50} \\ 2b - 5c = 0, & & \\ -15a = 3, & a = -\frac{3}{15} = -\frac{1}{5} & = -\frac{3}{25} \end{cases}$$

$$c = \frac{2}{5} \cdot b = -\frac{2}{5} \cdot \frac{3}{25} = -\frac{6}{125}$$

$$\Rightarrow y_2 = \frac{1}{5}(-x^3 - \frac{3}{5}x^2 - \frac{6}{25}x)$$

$$2) y'' - 5y' = \sin 5x = \operatorname{Im}(e^{i5x})$$

$$\Rightarrow y_2 = a \cdot e^{i5x}, y_2' = 5ai e^{i5x}, y_2'' = -25ae^{i5x}$$

$$\Rightarrow -25a - 25ai = 1, a = -\frac{1}{25(1+i)}$$

$$= -\frac{1}{25} \cdot \frac{1}{2} \cdot (1-i) = -\frac{1}{50}(1-i) \Rightarrow$$

$$\Rightarrow y_2 = -\frac{1}{50}(1-i)(\cos 5x + i\sin 5x)$$

$$\Rightarrow \operatorname{Im}(y_2) = -\frac{1}{50}\cos 5x + \frac{1}{50}\sin 5x$$

$$\text{Answer: } y = y_0 + y_1 + \operatorname{Im}(y_2)$$

549

$$y'' - 2y' = 2e^x, y(1) = -1, y'(1) = 0$$

$$1) y'' - 2y' = 0, \lambda^2 - 2\lambda = 0, \lambda = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$\Rightarrow y_2 = a e^x, y_2' = a e^x \Rightarrow a - 2a = 2$$

$$y_0 = C_1 + C_2 e^{2x}$$

$$-a = 2,$$

$$a = -2$$

$$y = C_1 + C_2 e^{2x} + 2e^x$$

$$y' = 2C_2 e^{2x} + 2e^x, y'(1) = 2C_2 e^2 + 2e = 0, C_2 = -\frac{1}{2}e$$

$$y(1) = C_1 + C_2 e^2 + 2e = -1$$

$$C_1 + e - 2e = -1$$

$$C_1 = -1 + e$$

$$y = -1 + e + e^{2x-1} - 2e^x$$

596

$$x^2 y'' - 3xy' + 5y = 3x^2$$

$$1) x = e^t, \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{dy}{dt} \cdot \frac{1}{e^t} = \ddot{y} e^{-t}$$

$$dx = e^t dt$$

$$y''_{xx} = e^{-2t}(\ddot{y} - \dot{y}) \Rightarrow \ddot{y} - \dot{y} - 3\dot{y} + 5y = 0$$

$$\ddot{y} - 4\dot{y} + 5y = 0, \lambda^2 - 4\lambda + 5 = 0,$$

$$\Delta = 16 - 20 = -4$$

$$\lambda_{1,2} = \frac{4 \pm 2i}{2} = 2 \pm i$$

$$y_0 = e^{2t}(C_1 \sin 2t + C_2 \cos 2t)$$

$$2) y_2 = a \cdot e^{2t}, y_2' = 2ae^{2t}, y_2'' = 4ae^{2t}$$

$$4e^{2t} - 4 \cdot 2ae^{2t} + 5ae^{2t} = 3e^{2t}$$

$$4a - 8a + 5a = 3, a = 3 \Rightarrow y_2 = 3e^{2t} = 3x^2$$

$$y = 3x^2 + x^2(C_1 \sin 2 \ln|x| + C_2 \cos 2 \ln|x|)$$

598

$$x^2 y'' - 2y = \sin \ln x$$

$$x = e^t, y'_x = \dot{y} e^{-t}, y''_{xx} = e^{-2t}(\ddot{y} - \dot{y})$$

$$1) \ddot{y} - \dot{y} - 2y = 0, 1 = \ln t \Rightarrow \operatorname{Im}(e^{it})$$

$$\lambda^2 - \lambda - 2 = 0, \Delta = 1 + 8 = 9, \lambda_{1,2} = \frac{1 \pm 3}{2} =$$

$$y_0 = C_1 e^t + C_2 e^{-t}$$

$$= 2i - 1$$

$$2) y_2 = a \cdot e^{it}, y_2' = ai e^{it}, y_2'' = -ae^{it}$$

$$\Rightarrow -a - ai - 2a = 1, a = \frac{-1}{3+i} = -\frac{1}{10}(3-i)$$

$$\Rightarrow y = C_1 e^t + C_2 e^{-t} + \frac{1}{10}(\cos t - 3 \sin t)$$

$$= C_1 x^2 + C_2 x^{-1} + \frac{1}{10}(\cos \ln|x| - 3 \sin \ln|x|)$$

5575

$$y'' - 2y' + y = \frac{e^x}{x},$$

$$1) \lambda^2 - 2\lambda + 1 = 0, (\lambda - 1)^2 = 0$$

$$\lambda = 1 \text{ к. 2}$$

$$y_0 = (c_1 + c_2 x) e^x = c_1 e^x + c_2 x e^x$$

$$\begin{cases} c_1 e^x + c_2 x e^x = 0 \\ c_1 e^x + c_2 x e^x (1+x) = \frac{e^x}{x} \end{cases}$$

$$c_1 e^x + c_2 x e^x (1+x) = \frac{e^x}{x}$$

$$c_1' = \frac{e^x}{x} \cdot \frac{1}{e^x}, c_1' = \frac{1}{x} = -1$$

$$2) c_1 = -x + \tilde{c}_1, c_2 = \ln x + \tilde{c}_2$$

$$\Rightarrow y = (-x + \tilde{c}_1 + (\ln x + \tilde{c}_2)x) e^x.$$

5572

$$y'' + y = \frac{1}{\sin x}$$

$$1) \lambda^2 + 1 = 0, \lambda = \pm i;$$

$$y_0 = c_1 \cos x + c_2 \sin x;$$

$$2) \begin{cases} c_1' \cos x + c_2' \sin x = 0 \quad | \cdot \cos x \\ -c_1' \sin x + c_2' \cos x = \frac{1}{\sin x} \end{cases}$$

$$\begin{pmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{pmatrix} \begin{pmatrix} c_1' \\ c_2' \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{\sin x} \end{pmatrix}$$

$$\det \begin{pmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{pmatrix} = \cos^2 x + \sin^2 x = 1, c_1' = 0$$

$$c_1' = -\frac{1}{\sin x};$$

$$c_2' = -\frac{c_1' \cos x}{\sin x} = + \frac{\cos x}{\sin^2 x}$$

$$2) c_1 = -\int \frac{1}{\sin x} dx = \int \frac{dx}{1 - \cos^2 x} =$$

$$= -\ln |\csc x + \cot x| + \tilde{c}_1$$

$$c_2 = \int \frac{dx}{\sin^2 x} = -\frac{1}{\sin x} + \tilde{c}_2$$

$$\Rightarrow y = (-\ln |\csc x + \cot x| + c_1) \cos x + (-\frac{1}{\sin x} + c_2) \sin x$$

$$c_1' = -1, c_2' = \csc x$$

$$c_1 = -x + \tilde{c}_1, c_2 = \int \frac{dx}{\sin x} = \ln |\tan x| + \tilde{c}_2$$

$$\Rightarrow y = (c_1 + \ln |\tan x|) \cos x + (c_2 - x) \sin x.$$

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581

$$x^3(y'' - y) = x^2 - 2 \quad | : x^3$$

$$y'' - y = \frac{1}{x} - \frac{2}{x^3}$$

$$1) \lambda = \pm 1, y_0 = c_1 e^x + c_2 e^{-x}$$

$$2) \begin{cases} c_1 e^x + c_2 e^{-x} = 0 \\ c_1 e^x - c_2 e^{-x} = \frac{1}{x} - \frac{2}{x^3} \end{cases}$$

$$\Rightarrow c_1' = \frac{1}{2e^x} \left(\frac{1}{x} - \frac{2}{x^3} \right)$$

$$c_2' = \frac{2-x^3}{2e^{-x}x^3} \Rightarrow c_2 = \frac{1}{2} \int \frac{1}{xe^x} dx - \int \frac{1}{xe^x} dx =$$

$$= \frac{1}{2} \left(\int \frac{1}{x} de^{-x} + \int \frac{1}{x^3} de^{-x} \right) =$$

$$-\frac{1}{2} e^{-x} + \int e^{-x} \frac{1}{x^3} dx =$$

597

$$x^2 y'' - 6y = 5x^3 + 8x^2$$

$$x = e^t, y' = \dot{y} e^{-t}, y'' = (\ddot{y} - \dot{y}) e^{-2t} \quad \left| \ddot{y} - \dot{y} - 6y = 5e^{3t} + 8e^{2t} \right.$$

$$\ddot{y} - \dot{y} - 6y = 0, \lambda^2 - \lambda - 6 = 0, \lambda = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

$$y_0 = c_1 e^{3t} + c_2 e^{-2t}$$

$$\begin{cases} 3c_1 e^{-2t} + c_2 e^{3t} = 0 \\ -2c_1 e^{-2t} + 3c_2 e^{3t} = 5e^{3t} + 8e^{2t} \end{cases}$$

$$+ : c_2 = \frac{1}{5} - \frac{8}{5} e^{-t} + c_2$$

$$c_1 = -\frac{1}{5} e^{5t} - \frac{2}{5} e^{4t} + \tilde{c}_1$$

$$y = -\frac{1}{5} e^{5t} - \frac{2}{5} e^{4t} + \tilde{c}_1 e^{-2t} + \tilde{c}_2 e^{3t} - \frac{8}{5} e^{2t} + \tilde{c}_2 e^{3t}$$

$$y = -\frac{1}{5} x^5 - 2x^4 + x^3 \ln |x| + c_1 x^{-2} + c_2 x^3.$$

$$y_p = \frac{1}{5} x^5 - 2x^4 + x^3 \ln |x| + c_1 x^{-2} + c_2 x^3.$$

$$y_p = \frac{1}{5} x^5 - 2x^4 + x^3 \ln |x| + c_1 x^{-2} + c_2 x^3.$$

645

$$4x, 2x+3, 6x+8 \text{ и.г. ?}$$

$$c_0(4x) + c_1(2x+3) + c_2(6x+8) = 0$$

$$x(-c_1 + 2c_2 + 6c_3) + (4c_0 + x_2 + 8c_3) = 0.$$

$\Rightarrow 1. f.$

648

$e^x, e^{2x}, e^{3x};$

$$\Delta = \begin{vmatrix} e^x & e^{2x} & e^{3x} \\ e^x & 2e^{2x} & 3e^{3x} \\ e^x & 4e^{2x} & 9e^{3x} \end{vmatrix} = e^{6x} \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{vmatrix} = e^{6x} \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 3 & 8 \end{vmatrix} \neq 0.$$

650

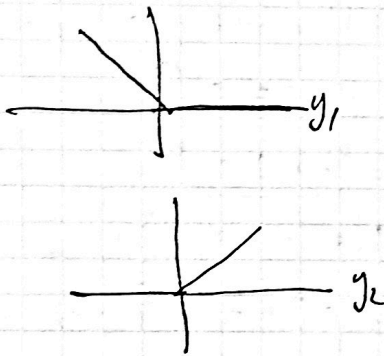
$1, \sin^2 x, \cos 2x, \cos 2x = 1 - 2\sin^2 x.$

665

$\Delta > 0, y_1 \rightarrow y_2$

Рисунок 1

$\Delta < 0$
no $\Delta > 0$.



666

$\Delta ?$ а) $y_1 \rightarrow y_2 - 1.2$. б) $\Delta = 0$
в) $y_2 \rightarrow y_1 - 1.2$. г) $\Delta < 0$ и $y_1 \rightarrow y_2$.

667

$y_1 = x, y_2 = x^5, y_3 = |x^5|$ - пер.

Зад. н.з. ? на E-1.13.

$x^2 y'' - 5xy' + 5y = 0$
н.к.д. на $(-2, +2)$
 $\neq 0$ на $\Delta \Rightarrow 2$ н.з. пер. л.

674

$1, \cos x; y_1 = 1, y_2 = \cos x$

$$\begin{vmatrix} 1 & \cos x & y \\ 0 & -\sin x & y' \\ 0 & -\cos x & y'' \end{vmatrix} = 0, \quad y' \cos x - y'' \sin x = 0.$$

682

$$x^2(x+1)y'' - 2y = 0, \quad \left(\frac{y}{y_2}\right)' = \frac{C_1 e^{-\frac{1}{x}}}{y_2^2}$$

$$a_0 = x^2(x+1), \quad a_1 = 0$$

$$\left(\frac{y}{y_2}\right)' = \frac{C_1}{y_2^2}, \quad \frac{y}{y_2} = \int \frac{C_1}{(1+\frac{1}{x})^2} \frac{1}{x^2} dx = \int \frac{C_1 x^2}{(x+1)^2} dx = C_1 \int \frac{x^2+1}{(x+1)^2} dx + C_2 \int \frac{1}{(x+1)^2} dx$$

$$y = (1+\frac{1}{x})C_1(x-2\ln(x+1)-\frac{1}{1+x}) + C_2(1+\frac{1}{x}).$$

682

$$(e^x+1)y'' - 2y' - e^x y = 0, \quad y_1 = e^x - 1, \quad a_0 = e^x + 1, \quad a_1 = -2$$

$$\left(\frac{y}{y_2}\right)' = \frac{C_1 e^{-2x}}{(e^x+1)^2} = \frac{C_1 e^{2x}}{(e^x-1)^2}$$

$$\Rightarrow \frac{y}{y_2} = C_1 \int \frac{e^{2x}}{(e^x-1)^2} dx = C_1 \int \frac{dz}{(z-1)^2} = \frac{C_1}{z-1} + C_2$$

$$y = (e^x-1) \frac{C_1}{e^x-1} + C_2 = C_1 + C_2(e^x-1)$$

686

$$x(x^2+6)y'' - 4(x^2+3)y' + 6xy = 0, \quad y = e^{ax}, \quad x(x^2+6)a^2 - 4(x^2+3)a + 6x = 0 \Rightarrow \text{не проходим.}$$

$$2) y = x^n, \text{ на } x=0: 3+n-2, 1+n-1, 2n-1, \dots, n-1$$

$$\begin{aligned} a) y'' &= ax^3 + bx + c, \\ y' &= 2ax + b \\ y'' &= 2a \end{aligned}$$

Втор. равно ст. сечение коэф.:

2a-8a

$$x(x^2+6)(2a-4(x^2+6)) + (2ax+b) + 6(ax^3+bx+c) = 0$$

$$\begin{aligned} 2b^2 &= 0 \\ c &= 2a \end{aligned} \Rightarrow y = x^2 + 2$$

$$\begin{aligned} \delta) y &= ax^3 + bx^2 + cx + d, \\ y' &= 3ax^2 + 2bx + c \\ y'' &= 6ax + 2b \end{aligned}$$

$$x(x^2+6)(6ax+2b) - 4(x^2+6)(3ax^2+2bx+c) + (3ax^2+2bx+c) = 0$$

$$\Rightarrow \begin{aligned} 2c &= 0 \quad c=0 \\ 6a-12b &= 0 \quad a=2b \Rightarrow a=1 \end{aligned}$$

$$y_2 = x^3$$

$$\Rightarrow c_1(x^2+2) + c_2x^3 = 0.$$

Д/р

8595

$$x^3 y'' - 2xy' = 6 \ln x$$

$$x^2 y'' - 2y' = 6 \frac{\ln x}{x}$$

$$x = e^t \Rightarrow y' = \dot{y} \cdot e^{-t}; y'' = e^{-2t}(\ddot{y} - \dot{y})$$

$$\Rightarrow \ddot{y} - \dot{y} - 2y = 6 \frac{t}{e^t} = 6e^{-t}t$$

$$1) \ddot{y} - \dot{y} - 2y = 0$$

$$\lambda^2 - \lambda - 2 = 0, \Delta = 1+8=9$$

$$\lambda_{1,2} = \frac{1 \pm 3}{2} = 2; -1$$

$$\Rightarrow y_0(t) = c_1 e^{2t} + c_2 e^{-t}$$

$$2) \int c_1 e^{2t} + c_2 e^{-t} = 0$$

$$2c_1 e^{2t} - c_2 e^{-t} = 6e^{-t}t$$

$$\% c_1 e^{2t} = 6e^{-t}t \quad \left| \cdot e^{-2t} \right| \quad 2c_1 = 6e^{-3t}t$$

$$c_1 = 3t \cdot e^{-3t}; \quad \left| \cdot e^{3t} \right| \quad 2c_1 = 6t$$

$$c_1 = \frac{3}{2}t + d(e^{-3t})$$

$$c_2 = -\frac{1}{2}t + \tilde{c}_2$$

$$\begin{aligned} c_2 &= -\frac{1}{2}t + \tilde{c}_2 \\ c_2 &= -\frac{1}{2}t + \tilde{c}_2 \end{aligned}$$

$$\begin{aligned} C_1 &= -\frac{4}{3} \left(t \cdot e^{-3t} + \frac{1}{3} \int e^{-3t} (4t-2) dt \right) \\ &= -\frac{4}{3} e^{-3t} \left(t + \frac{1}{3} \frac{t^2}{2} + \frac{2}{9} e^{3t} \right) \end{aligned}$$

$$\Rightarrow y = -\frac{4}{3} e^{3t} \left(t + \frac{1}{3} \frac{t^2}{2} + \frac{2}{9} \right) + \left(-\frac{t^2}{2} + \tilde{c}_2 \right) e^{-t} =$$

$$= -\frac{4}{3} e^{-t} \left(t + \frac{1}{3} e^{3t} \tilde{c}_1 \right) + \left(-\frac{t^2}{2} + \tilde{c}_2 \right) e^{-t}$$

$$y = e^{-t} \left(-\frac{t^2}{2} + \tilde{c}_2 - \frac{2}{3} t + \frac{e^{3t}}{3} \tilde{c}_1 \right) =$$

$$= \frac{1}{x} \left(-\frac{\ln^2 x}{2} + \frac{1}{3} \ln x - \frac{1}{9} - \frac{1}{3} x^3 \tilde{c}_1 \right) =$$

$$= c_1 x^2 + \frac{1}{x} \left(c_2 - \frac{2}{3} \ln x - \frac{\ln^2 x}{2} \right)$$

8657

$$y'' + 2y' + y = x e^x + \frac{1}{x e^x}$$

$$1) \frac{1}{x} x^2 + 2x + 1 = 0 \quad (d+1)^2 = 0$$

$$y_0 = (c_1 + c_2 x) e^{-x} \quad \lambda = -1 \text{ (пр. 2)}$$

$$\ominus c_1 e^{-x} + c_2 x e^{-x}$$

$$\begin{cases} c_1' e^{-x} + c_1 x e^{-x} = 0 \\ -c_1' e^{-x} + c_2' e^{-x} (1-x) = x e^x + \frac{1}{x e^x} \end{cases}$$

$$\begin{aligned} e^x c_2' (1-x) &= x e^x + \frac{1}{x e^x} \\ c_2' &= x e^{2x} + \frac{1}{x} \end{aligned} \quad \left| \begin{aligned} c_1' &= -x c_2' \\ c_1' &= -x^2 e^{2x} + 1 \end{aligned} \right.$$

$$\begin{aligned} c_1 &= -\int x^2 e^{2x} dx - x \\ &= \frac{1}{2} x^2 d(e^{2x}) = \frac{1}{2} (x^2 e^{2x} - 2 \int e^{2x} x dx) \end{aligned}$$

$$\begin{aligned} \frac{1}{2} \int x dx d(e^{2x}) &= \\ &= \frac{1}{2} (x e^{2x} - \frac{1}{2} \int e^{2x} dx) \end{aligned}$$

$$\frac{1}{2} (x e^{2x} - \frac{1}{2} e^{2x}) + \tilde{c}_1$$

$$\frac{1}{2} (x^2 e^{2x} - e^{2x} (x - \frac{1}{2})) + \tilde{c}_1$$

$$c_1 = -\frac{1}{2} e^{2x} (x^2 x + \frac{1}{2}) - x + \tilde{c}_1$$

$$c_2 = \ln|x| + \frac{1}{2} e^x (x - \frac{1}{2}) + \tilde{c}_2$$

$$\Rightarrow y = -\frac{1}{2} e^x (x^2 - x + \frac{1}{2}) - x e^{-x} + c_1 e^{-x}$$

$$+ x e^{-x} \ln|x| + \frac{1}{2} x e^x (x - \frac{1}{2}) + x e^{-x} c_2 =$$

$$= (c_1 + c_2 x + x \ln|x|) e^{-x} + \frac{x-1}{4} e^x.$$

SLES

$$x^2 y'' - 2y = \frac{2x^2}{x+1}$$

$$x = e^t, \quad y'' = e^{-4t} (y'' - y')$$

$$y'' - y' - 2y = \frac{3e^{2t}}{e^t + 1}$$

$$1) \lambda^2 - \lambda - 2 = 0 \quad \lambda = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$y_0 = c_1 e^{-t} + c_2 e^{2t}$$

$$c_1 e^{-t} + c_2 e^{2t} = 0$$

$$-c_1 e^{-t} + 2c_2 e^{2t} = \frac{3e^{2t}}{e^t + 1}$$

$$3e^{2t} c_2' = \frac{3e^{2t}}{e^t + 1} \quad c_2' = \frac{1}{e^t + 1}$$

$$c_1' = -c_2' e^{3t} = -\frac{e^{3t}}{e^t + 1}$$

$$c_1 = -\int \frac{e^{3t}}{e^t + 1} dt = -\int \frac{e^{2t}}{e^t + 1} dt = -\int \frac{e^{2t}}{e^t + 1} dt$$

$$= -\int \frac{e^{2t}}{e^t + 1} \cdot \frac{1}{e^t} dt = -\int \frac{e^t}{e^t + 1} dz =$$

$$= -\left(\int \frac{(z+1)^2}{z+1} dz - \int \frac{2z}{z+1} dz + \int \frac{1}{z+1} dz \right)$$

$$\int \frac{(z+1)^2}{z+1} dz = \int (z+1) dz = \frac{z^2}{2} + z$$

$$-2 \int \frac{2z}{z+1} dz = -2 \int \frac{2(z+1)-2}{z+1} dz$$

$$= -2 \int \left(2 - \frac{2}{z+1} \right) dz = -4z + 4 \ln|z+1|$$

$$-2z + 2 \ln|z+1|$$

$$= \frac{z^2}{2} - 2z + 2 \ln|z+1| = \ln|z+1| + \tilde{c}_1$$

$$= \frac{z^2}{2} - 2z + \ln|z+1| + \tilde{c}_1 = \frac{e^{2t}}{2} - e^t + \ln(e^t + 1) + \tilde{c}_1$$

$$c_2 = \int \frac{1}{e^t + 1} dt = \int \frac{e^t - 1}{e^t(e^t + 1)} dt = \int \frac{1}{e^t + 1} dt =$$

$$= \int \left(\frac{1}{e^t + 1} - \frac{1}{e^t} \right) dt = \ln|e^t + 1| - \ln|e^t| + \tilde{c}_2$$

$$= t - \ln|(e^t + 1)| + \tilde{c}_2$$

$$\Rightarrow y = \frac{1}{2} e^t + 1 + e^t \ln|(e^t + 1)| + \tilde{c}_1 e^{-t}$$

$$+ t e^{2t} - e^{2t} \ln|(e^t + 1)| + \tilde{c}_2 e^{2t} =$$

$$= x^2 \ln \frac{e^x}{x+1} - \frac{x}{2} + 1 - \frac{1}{x} \ln|e^x + 1|$$

SLES

$$x^2 y'' - x y' + y = \frac{\ln x}{x} + \frac{x}{\ln x}$$

$$x = e^t, \quad y'' = e^{-2t} (y'' - y')$$

$$y'' - y' - y = \frac{t}{e^t} + \frac{e^t}{t} = t e^{-t} + t^{-1} e^t$$

$$1) \lambda^2 - 2\lambda + 1 = 0, \quad (\lambda - 1)^2 = 0; \quad \lambda = 1 \text{ (double)}$$

$$y_0 = (c_1 + c_2 x) e^t; \quad y_0 = c_1 e^t + c_2 t e^t$$

$$c_1 e^t + c_2 t e^t = 0$$

$$c_1 e^t + c_2 e^t (1+t) = t e^t + t^{-1} e^t$$

$$c_2 e^t = t e^t + t^{-1} e^t, \quad c_2' = e^{-t} + t^{-2}$$

$$c_1' = -c_2' = -e^{-t} - t^{-1}$$

$$c_2 = -\frac{1}{2} e^{-2t} + t^{-1} + \tilde{c}_2$$

$$c_1 = -\ln|t| + \int \frac{1}{2} t d(e^{-2t})$$

$$= -\ln|t| + \frac{1}{2} \int \frac{1}{t} dt = -\ln|t| + \frac{1}{2} \ln|t| + \tilde{c}_1$$

$$= -\frac{1}{2} \ln|t| + \frac{1}{2} \ln|t| + \tilde{c}_1 = \tilde{c}_1$$

$$= \frac{1}{2} e^{-2t} (t + \frac{1}{2}) + \tilde{c}_1$$

$$\Rightarrow -\ln|t| + \frac{1}{2} e^{-2t} (t + \frac{1}{2}) + \tilde{c}_1$$

$$y = -e^t \ln|t| + \frac{1}{2} e^{-t} (t + \frac{1}{2}) + e^t \tilde{c}_1$$

$$- \frac{1}{2} t e^{-t} - e^t + \tilde{c}_2 t e^t = e^{-t} \ln|t| + \frac{1}{2} e^{-t} (t + \frac{1}{2}) + e^t \tilde{c}_1$$

$$+ e^t (c_1 \ln|t| + c_2 t) = \frac{1}{4} e^{-t} + e^t (-\ln|t| + c_2 t + c_1)$$

$$= \frac{1}{4} \cdot \frac{1}{x} + x (-\ln|\ln x| + c_2 \ln|x| + c_1)$$

646

$$x^2 + 2, 3x^2 - 1, x + 4$$

$$\begin{vmatrix} x^2 + 2 & 3x^2 - 1 & x + 4 \\ 2x & 6x & 1 \\ 2 & 6 & 1 \end{vmatrix}$$

$$c_1(x^2 + 2) + c_2(3x^2 - 1) + c_3(x + 4) = 0$$

$$x^2(c_1 + 3c_2) + x(c_3) + (2c_1 - c_2 + 4c_3) = 0$$

$$\begin{cases} c_3 = 0 \\ c_1 + 3c_2 = 0; c_1 = -3c_2 \\ 2c_1 - c_2 + 4c_3 = 0, -6c_2 - c_2 = 0 \end{cases} = 0$$

$\Rightarrow$ n.r.

651

$$\sin x, \cos x, 2 + e^x$$

$$\begin{aligned} & \frac{e^x + e^{-x}}{2} \cdot \frac{e^x + e^{-x}}{2} \cdot \frac{e^x + e^{-x}}{2} \\ & c_1 \sin x + c_2 \cos x + c_3(2 + \sin x + \cos x) = 0 \\ & \sin x(c_1 + c_3) + \cos x(c_2 + c_3) + 2c_3 = 0 \end{aligned}$$

B

$$\begin{vmatrix} \sin x & \cos x & 2 + e^x \\ \cos x & \sin x & e^x \\ \sin x & \cos x & e^x \end{vmatrix} = \sin^2 x e^x +$$

$$\begin{aligned} & + \cos x \cdot \sin x e^x + \cos^2 x (2 + e^x) - \\ & \sin^2 x (2 + e^x) - \sin x \cos x e^x - \cos^2 x e^x = \\ & = 2(\cos^2 x - \sin^2 x) = 2 \neq 0 \Rightarrow \text{n.r.} \end{aligned}$$

654

$$\sin x, \cos x, 2e^x - 1, 3e^x + 5$$

$$\begin{vmatrix} \sin x & \cos x & 2e^x - 1 & 3e^x + 5 \\ \cos x & \sin x & 2e^x & 3e^x \\ \sin x & \cos x & 2e^x & 3e^x \\ \cos x & \sin x & 2e^x & 3e^x \end{vmatrix} \neq 0$$

n.r. 2 eigene. 6 p. n.r.

675

$$x, e^x;$$

$$\begin{vmatrix} x & e^x & y \\ 1 & x & y' \\ 0 & e^x & y'' \end{vmatrix} = 0 \Rightarrow y''(x-1) + y' + y = 0$$

$$\begin{vmatrix} x & e^x \\ 1 & x \end{vmatrix}$$

677

$$x^2 - 3x, 2x^2 + 9, 2x + 1$$

$$a \quad b \quad c$$

$$b = 2a + 3c \Rightarrow$$

$$\begin{vmatrix} x^2 - 3x & 2x + 3 & y \\ 2x - 3 & 2 & y' \\ 2 & 0 & y'' \end{vmatrix} = 0 \Rightarrow y''(-2x^2 - 6x + 9) + (y')^2(2x + 3) + 4y = 0$$

678

$$e^x, \sin x, \cos x$$

$$\begin{vmatrix} e^x & \sin x & \cos x \\ \frac{e^x}{e^x} & \frac{\sin x}{\sin x} & \frac{\cos x}{\cos x} \\ \frac{e^x}{e^x} & \frac{\sin x}{\sin x} & \frac{\cos x}{\cos x} \end{vmatrix} \Rightarrow y'' - y = 0$$

681

$$(2x+1)y'' + 4xy' + 4y = 0$$

$$y_1 = e^{ax}, y_1' = ae^{ax}, y_1'' = a^2 e^{ax}$$

$$a^2(2x+1) + 4xa - 4 = 0$$

$$x(2a^2 + 4a) + a^2 - 4 = 0$$

$$a = -2 \quad y_1 = e^{-2x}$$

$$\int \frac{1}{2x+1} dx = \int \frac{1}{2x+1} dx = \frac{1}{2} \ln|2x+1| + C$$

$$\frac{2x+1}{2x+1} = 2 \int \frac{2x+1-1}{2x+1} dx = 2 \left(x - \frac{1}{2} \int \frac{1}{2x+1} dx \right)$$

$$= 2x - \ln|2x+1| + C_0$$

$$\left(\frac{y}{y_1} \right)' = \frac{y y_1' - y_1 y'}{y_1^2}; \quad \frac{y}{y_1} = C_1 \int \frac{e^{-2x}}{e^{-4x}} dx =$$

$$= C_1 \int e^{2x} dx = C_1 \int e^{2x} dx = C_1 \frac{1}{2} e^{2x} = C_1 e^{2x}$$

$$= C_1 \left(\frac{1}{2} e^{2x} + 2 \int x e^{2x} dx \right)$$

$$\frac{1}{2} e^{2x} + 2 \left(\int x e^{2x} dx \right) = \frac{1}{2} e^{2x} + x e^{2x} - \frac{1}{2} e^{2x} = x e^{2x}$$

$\frac{y}{y_1} = C_1 x e^{2x} + C_2 \cdot C$
 $y = C_1 x + \frac{C_2 \cdot C}{C_2} e^{-4x}$
 $\int \frac{1}{x^2} dx = -\frac{1}{x} + C$
 $x y'' + C y' - x y = 0, y_1 = \frac{e^x}{x}$
 $\left(\frac{y}{y_1}\right)' = \frac{C y'}{y_1^2}$
 $\int \frac{1}{x^2} dx = -\frac{1}{x} + C$
 $\frac{y}{y_1} = C_1 \int \frac{e^{2x}}{x^2} \cdot e^{-2 \ln(x) + C} = C_1 \int \frac{e^{2x}}{x^2} \cdot \frac{1}{x^2} dx$
 $y' + \frac{2}{x} y' - y = 0$
 $\left(\frac{y}{y_1}\right)' = \frac{C_1}{e^{2x}} \Rightarrow \frac{y}{y_1} = -\frac{1}{2} C_1 e^{-2x} + C_2$
 $y = -\frac{1}{2} C_1 \cdot \frac{e^{-x}}{x} + \frac{e^x}{x} C_2 = \frac{1}{x} (-\frac{1}{2} e^{-x} + e^x C_2)$

$\int \frac{1}{x^2} dx = -\frac{1}{x} + C$
 $y'' - 2(1 + \ln x) y = 0, y_1 = \ln x$
 $\left(\frac{y}{y_1}\right)' = \frac{C_1}{\ln x} \Rightarrow \frac{y}{y_1} = -C_1 \ln x - C_2 x + C_2$
 $y = C_2 (\ln x + C_1 (1 + x \ln x))$
 $x(x+4)y'' - (2x+4)y' + 2y = 0$
 $y'' - \frac{(2x+4)}{x(x+4)} y' + \frac{2}{x(x+4)} y = 0$
 $\int \frac{1}{x(x+4)} dx = -\int \frac{1}{x(x+4)} dx = -2 \int \frac{x+2}{x(x+4)} dx$
 $= -2 \int \frac{x+4-2}{x(x+4)} dx =$
 $= -2 \int \frac{1}{x} dx + 2 \int \frac{1}{x+4} dx = 2 \ln \left| \frac{x+4}{x} \right| + C$
 $\ln|x+4| - \ln|x|$
 $= -2 \ln|x| + \ln|x+4| - \ln|x+4| = -\ln|x(x+4)|$
 $y_{x1} = e^{2x}; y_{x2} = \ln x + T$
 $y = ax^2 + bx + c, y' = 2ax + b, y'' = 2a$
 $n(n-1) - 2(n+1) + 2 = 0, n^2 - 3n + 2 = 0$
 $\Delta = 1, n = \frac{3 \pm 1}{2}$

$y = ax^2 + bx + c, y' = 2ax + b, y'' = 2a$
 $(x^2 + 4x) \cdot 2a - (2x + 4)(2ax + b) + 2y = 0$
 $x^2(2a - 4a) + x(8a - 2b + 8a) - 4b + 2c = 0$
 $2a = 2b$
 $8a - 2b = 0$
 $4a = b$
 $y = (x+2)2(x)$
 $\Rightarrow (x^2 + 6x + 4x) - 8x = 0$
 $2x^2 = 8x$
 $2x = 4$
 $x = 2$
 $\frac{1}{8} \frac{df}{f} = \frac{dx}{x^3 + 6x^2 + 6x}$

Kiri padoxa
 $\int \frac{1}{x^2} dx = -\frac{1}{x} + C$
 $\begin{cases} \dot{x} = 2x + y \\ \dot{y} = 3x + 4y \end{cases} \quad A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$
 $|A - \lambda I| = (2 - \lambda)(4 - \lambda) - 3 = (\lambda - 1)(\lambda - 5)$
 $\lambda_1 = 1: A - \lambda_1 I = \begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix} e_1 = (1, -1)$
 $\lambda_2 = 5: \begin{bmatrix} -3 & 1 \\ 3 & -1 \end{bmatrix} e_2 = (1, 3)$
 $\begin{pmatrix} x \\ y \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^t + C_2 \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{5t}$
 $\begin{cases} x = C_1 e^t + C_2 e^{5t} \\ y = -C_1 e^t + 3C_2 e^{5t} \end{cases}$

$\int \frac{1}{x^2} dx = -\frac{1}{x} + C$
 $\begin{cases} \dot{x} = x - y + z \\ \dot{y} = x + y - z \\ \dot{z} = 2x - y \end{cases} \quad \begin{matrix} \lambda_1 = 1 \\ \lambda_2 = 1 \\ \lambda_3 = -1 \end{matrix}$
 $A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ 2 & -1 & 0 \end{pmatrix} \quad \lambda_1: A - \lambda_1 I = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ 2 & -1 & -1 \end{pmatrix}$
 $C_1 = (1, 1, 1)$
 $\lambda_2: A - \lambda_2 I = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ 2 & -1 & -1 \end{pmatrix} \Rightarrow C_2 = (1, 0, 1)$
 $\begin{pmatrix} 1 & 0 & -1 \\ 2 & -1 & -1 \end{pmatrix}$

$$\lambda_1: A - \lambda_1 I = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 2 & -1 & 1 \end{pmatrix} \quad e_1(1, -1, -1)$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} e^{2t} + c_3 \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} e^{-t}$$

801

$$\begin{cases} \dot{x} = x - y - z \\ \dot{y} = x + y \\ \dot{z} = 3x + z \end{cases} \quad A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

$$\lambda_1 = 1 \quad A - \lambda_1 I = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & 0 \\ 3 & 0 & 0 \end{bmatrix}$$

$$\lambda_2 = 1 \pm 2i \quad e_1(0, 1, -1)$$

$$\lambda_2 = 1 + 2i$$

$$A - \lambda_2 I = \begin{bmatrix} -2i & -1 & -1 \\ 1 & -2i & 0 \\ 3 & 0 & -2i \end{bmatrix}$$

$$e_2(2i, 1, 3); \begin{pmatrix} 2i \\ 1 \\ 3 \end{pmatrix} e^{(1+2i)t} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} e^t + \begin{pmatrix} 2i \\ 0 \\ 0 \end{pmatrix} e^{2it}$$

$$\cdot e^{t(\cos 2t + i \sin 2t)} =$$

$$= \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} \cos 2t + \begin{pmatrix} -2i \\ 0 \\ 0 \end{pmatrix} \sin 2t e^t + i \left(\begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} \cos 2t + \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} \sin 2t \right) e^t$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = c_1 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} e^t + c_2 e^t \begin{pmatrix} -2 \sin 2t \\ \cos 2t \\ 3 \cos 2t \end{pmatrix} + c_3 e^t \begin{pmatrix} 2 \cos 2t \\ \sin 2t \\ 3 \sin 2t \end{pmatrix}$$

805

$$\dot{x} = Ax \quad A = \begin{bmatrix} -1 & -2 & 2 \\ -2 & -1 & 2 \\ -3 & -2 & 3 \end{bmatrix}$$

$$\lambda_1 = 1$$

$$\lambda_2 = \pm i \quad A - \lambda_1 I = \begin{bmatrix} -2 & -2 & 2 \\ -2 & -2 & 2 \\ -3 & -2 & 2 \end{bmatrix}$$

$$e_1(0, 1, 1)$$

$$A - \lambda_1 I = \begin{bmatrix} -1-i & -2 & 2 \\ -2 & -1-i & 2 \\ -3 & -2 & 3-i \end{bmatrix} \sim \begin{bmatrix} -1-i & -1+i & 0 \\ -2 & -1+i & 2 \\ -1 & -1+i & 1 \end{bmatrix}$$

$$e_2(2, 2, 3+i),$$

$$e_2 \cdot e^{it} = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} + i \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} (\cos t + i \sin t) =$$

$$= \left(\begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} \cos t + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \sin t \right) +$$

$$i \left(\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \cos t + \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} \sin t \right)$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = c_1 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} e^t + c_2 \begin{pmatrix} 2 \cos t \\ 2 \cos t \\ 1 \cos t - 1 \sin t \end{pmatrix} + c_3 \begin{pmatrix} 2 \sin t \\ 2 \sin t \\ 1 \cos t + 1 \sin t \end{pmatrix}$$

$$\begin{cases} \dot{x} = x + z \\ \dot{y} = x + z \\ \dot{z} = x + y \end{cases} \quad \lambda_1 = 2 \quad k_1 = 1 \\ \lambda_2 = -1 \quad k_2 = 2$$

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$A - \lambda_1 I = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 2 & 3 & 0 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$$e_1(1, 1, 1), \lambda_2 = -1: \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$e_2 = (1, -1, 0), \quad e_3 = (0, 1, -1). \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} e^{2t} + c_2 \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} e^{-t} + c_3 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} e^{-t}$$

808

$$\begin{cases} \dot{x} = x - y + z \\ \dot{y} = x + y + z \\ \dot{z} = z - y \end{cases} \quad \lambda_1 = 2 \quad k_1 = 1 \\ \lambda_2 = 1 \quad k_2 = 1$$

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$\lambda_1: A - \lambda_1 I = 0 \Rightarrow e_1(1, 0, 1)$$

$$\lambda_2: x = (a + bt) e^t$$

$$y = (c + dt) e^t \quad k_2 = 2 \Rightarrow 1 \text{ cr.}$$

$$z = (e + ft) e^t$$

$$\begin{cases} a + b + t = a + b - c - dt + e + ft \\ c + 1 + t = a + b + c + dt - e - ft \\ k + nt + n = 2k + 2nt - c - dt \end{cases}$$

$$\begin{cases} d = n \\ b + c = k \\ p = n \\ d + k = a \\ n + c = k \end{cases} \quad \begin{matrix} d = b = n & n = c_1 \\ k = c + n & c = c_2 \\ a = 2n + c \end{matrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2c_1 + c_2 + t \\ c_2 + c_1 \\ c_1 + c_2 + c_1 t \end{pmatrix} e^t + c_3 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} e^{2t}$$

811

$$\begin{cases} \dot{x} = 2x - y - z \\ \dot{y} = 2x - y - 2z \\ \dot{z} = -x + y + 2z \end{cases}$$

$\lambda_1 = 1$ up.3.
 $A = \begin{bmatrix} 2 & -1 & -1 \\ 2 & -1 & 2 \\ -1 & 1 & 2 \end{bmatrix}$

2 n.u.j. $\Rightarrow$ num. n.u.j.

$$\begin{cases} x = (a+bt)e^t \\ y = (c+dt)e^t \\ z = (k+nt)e^t \end{cases}$$

$$\begin{aligned} a+b+bt &= 2a+2bt - c - dt - k+nt \\ c+d+dt &= 2a+2bt - c - dt - 2k - 2nt \\ k+n+nt &= -a - bt + c + dt + 2k + 2nt \end{aligned}$$

$$\begin{cases} b = d + n \\ b = a - c - k \\ c + d = 2a - c - 2k \\ n = -a + c + k \end{cases} \quad \begin{aligned} a &= c_1, c = c_2, k = c_3 \\ b &= c_1 - c_2 + c_3 \\ n &= -c_1 + c_2 + c_3 \\ d &= 2c_1 - 2c_2 - 2c_3 \end{aligned}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} c_1 + (c_1 - c_2 - c_3)t \\ c_2 \\ c_3 \end{pmatrix} e^t$$

812

$$\begin{cases} \dot{x} = 4x - y \\ \dot{y} = x + y - z \\ \dot{z} = x + z \end{cases}$$

$\lambda = 2$ up.3.
 $A = \begin{bmatrix} 4 & -1 & 0 \\ 1 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 2 & -1 & 0 \\ 1 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix} \quad \text{1 c. b.}$$

$\lambda = 2: B^2$
 $\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 1 & -1 & 1 \end{bmatrix} \quad x_1 - x_2 + x_3 = 0$

$e_1(1, 0, 0), e_2 = Be_1 = (2, 3, 1)$
 $e_3 = Be_2 = B^2 e_1 = (1, 2, 1)$

$$u_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} e^{2t}, \quad u_2 = \begin{pmatrix} 2t+t \\ 3+2t \\ 1+t \end{pmatrix} e^{2t}$$

$$u_3 = \begin{pmatrix} 1+2t+\frac{1}{2}t^2 \\ 0+3t+\frac{1}{2}t^2 \\ 0+t+\frac{1}{2}t^2 \end{pmatrix} e^{2t}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = c_1 u_1 + c_2 u_2 + c_3 u_3$$

865

$\dot{x} = Ax, \quad A = \begin{bmatrix} 4 & 2 & -2 \\ 1 & 3 & -1 \\ 3 & 3 & -1 \end{bmatrix} \quad \lambda = 2$ up.3.

$$\begin{cases} x = (a+bt)e^{2t} \\ y = (c+dt)e^{2t} \\ z = (k+nt)e^{2t} \end{cases} \quad \begin{aligned} 2a+2bt+b &= 4a+4bt+c+2d+2kt-2nt \\ 2c+2dt+d &= c+3kt-nt \end{aligned}$$

$$(2c+2nt+n) = 3a+3bt+3c+3kt-nt$$

$$\begin{cases} b+d=a \\ b=2d=3a \\ d=a+c-k \\ 2n=b+d \\ 3k+n=3a+3c \end{cases} \quad \begin{aligned} b &= 2d=3a \\ a &= c_1, b = c_2, k = c_3 \\ d &= c_1 + c_2 - c_3 \\ b &= 2c_1 + c_2 \end{aligned}$$

$$\Rightarrow \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

818

819

$$\begin{cases} \dot{x} = 2x + y \\ \dot{y} = 4y - x \end{cases} \quad A = \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 2-\lambda & 1 \\ -1 & 4-\lambda \end{bmatrix} \quad (2-\lambda)(4-\lambda) + 1 = 0$$

$$\lambda^2 - 6\lambda + 9 = 0 \Rightarrow (\lambda - 3)^2 = 0 \Rightarrow \lambda = 3 \text{ up.2}$$

$\lambda = 3: x = (a+bt)e^{3t}$
 $y = (c+dt)e^{3t}$

$$\Rightarrow \begin{cases} a+b+bt = 2a+2bt+c+dt \\ c+d+dt = -a-bt+4c+4dt \end{cases}$$

$$\begin{cases} b = 2b + d, \quad b = -d \\ a + b = 2a + c, \quad b = a + c \\ d = -b + 4d \Rightarrow d = 0 \Rightarrow b = 0 \Rightarrow c = 0 \\ c + d = -a + 4c \Rightarrow d = 0 \Rightarrow b = 0 \Rightarrow c = 0 \end{cases}$$

818

819

S298

$$\begin{cases} \dot{x} = 2x - y + z \\ \dot{y} = x + 4y - z \\ \dot{z} = x - y + 2z \end{cases} \quad \lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 2$$

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 4 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$\lambda_1: A - \lambda_1 I = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 1 \end{bmatrix} \Rightarrow e_1(0, 1, 1)$$

$$\lambda_2: A - \lambda_2 I = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 0 \end{bmatrix} \Rightarrow e_2(1, 1, 1)$$

$$\lambda_3: A - \lambda_3 I = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 0 \end{bmatrix} \Rightarrow e_3(1, 0, 1)$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = c_1 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} e^{2t} + c_3 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} e^{2t}$$

S299

$$\begin{cases} \dot{x} = 3x - y + z \\ \dot{y} = x + y + z \\ \dot{z} = 4x - y + 4z \end{cases} \quad \lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 5$$

$$A = \begin{bmatrix} 3 & -1 & 1 \\ 1 & 1 & 1 \\ 4 & -1 & 4 \end{bmatrix} \Rightarrow A - \lambda_1 I = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 0 & 1 \\ 4 & -1 & 3 \end{bmatrix}$$

$$\Rightarrow e_1(x, x, -x) \quad e_2(1, 1, -1)$$

$$A - \lambda_2 I = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ 4 & -1 & 2 \end{bmatrix} \Rightarrow e_2(1, 1, 1)$$

$$\begin{bmatrix} 0 & 3 & 2 \\ 1 & -1 & 1 \end{bmatrix}$$

$$A - \lambda_3 I = \begin{bmatrix} -2 & -1 & 1 \\ 1 & -4 & 1 \\ 4 & -1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & 1 \\ 0 & -3 & 3 \\ 0 & 15 & -5 \end{bmatrix}$$

$$\Rightarrow e_3(1, 1, 3)$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} e^t + c_2 \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} e^{2t} + c_3 \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} e^{5t}$$

S303

$$\begin{cases} \dot{x} = 2x + 2y \\ \dot{y} = x + 2z \\ \dot{z} = y - 2x - z \end{cases} \quad A = \begin{bmatrix} 2 & 2 & 0 \\ 1 & 0 & 2 \\ -2 & 1 & -1 \end{bmatrix}$$

$$\lambda_1 = 0, \lambda_2 = i, \lambda_3 = -i$$

$$A - \lambda_1 I = \begin{bmatrix} 2 & 2 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & -2 \end{bmatrix} \Rightarrow e_1(0, 2, 1)$$

$$A - \lambda_2 I = \begin{bmatrix} 2-i & 2 & 0 \\ 1 & -i & 2 \\ 2 & 1 & -1-i \end{bmatrix} \sim \begin{bmatrix} 1-i & -1-i & 0 \\ 1 & -i & 2 \\ 2 & 1 & -1-i \end{bmatrix}$$

$$\Rightarrow e_2(2, 2, i-1)$$

$$\Rightarrow \begin{pmatrix} 2 \\ 2 \\ i-1 \end{pmatrix} e^{it} = \left(\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} + i \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right) (\cos t + i \sin t)$$

$$\Rightarrow \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} \cos t + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \sin t + i \left(\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \cos t + \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} \sin t \right)$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = c_1 \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} e^t + c_2 \begin{pmatrix} 2 \cos t + 2 \sin t \\ 2 \cos t - \sin t \\ -\cos t - \sin t \end{pmatrix} + c_3 \begin{pmatrix} 2 \sin t \\ 2 \sin t \\ -\sin t \end{pmatrix}$$

$$+ c_3 \begin{pmatrix} \cos t \\ \sin t \\ \cos t - \sin t \end{pmatrix}$$

S358

$$\dot{x} = Ax, \quad A = \begin{pmatrix} -3 & 2 & 2 \\ -3 & -1 & 1 \\ -1 & 2 & 0 \end{pmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} -3-\lambda & 2 & 2 \\ -3 & -1-\lambda & 1 \\ -1 & 2 & -\lambda \end{vmatrix} = \begin{vmatrix} -(2+\lambda) & 0 & 2+\lambda \\ -3 & -1-\lambda & 1 \\ -1 & 2 & -\lambda \end{vmatrix}$$

$$= (2+\lambda) \begin{vmatrix} -1 & 0 & 1 \\ 0 & -1-\lambda & -2 \\ 0 & 2 & -\lambda-1 \end{vmatrix} = (2+\lambda) \begin{vmatrix} -1 & -2 \\ 2 & -\lambda-1 \end{vmatrix}$$

$$= 0 \Rightarrow \lambda = -2$$

$$\lambda_{2,3} =$$

$$(\lambda+1)^2 + 4 = 0$$

$$2(\lambda+1) = \pm 2i$$

$$\lambda = -1 \pm 2i$$

$$\lambda = -2: A - \lambda I = \begin{bmatrix} -1 & 2 & 2 \\ -3 & 1 & 1 \\ -1 & 2 & 2 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \Rightarrow e_1(1, -1, 1)$$

$$0 = 5 - 5$$

$$\lambda = -1+2i: A - \lambda I = \begin{bmatrix} -2-2i & 2 & 2 \\ -3 & -2i & 1 \\ -1 & 2 & 1-2i \end{bmatrix} \sim$$

$$\sim \begin{bmatrix} -2-2i & 2 & 0 \\ -3 & -2i & 1+2i \\ -1 & 2 & -1-2i \end{bmatrix} \Rightarrow y = \frac{2i-2}{2} x = (i-1)x$$

$$(2i-6)y + (4+8i)z = 0$$

$$z = \frac{6+2i}{4+8i} y = \frac{6+2i}{4+8i} y$$

$$= \frac{6+2i}{4+8i} y = \frac{6+2i}{4+8i} y$$

$$i-1 \Rightarrow$$

$$-2i+2-2+2i$$

$$-2i+2i \Rightarrow$$

$$e_2(i, 1, i) \Rightarrow$$

$$\begin{pmatrix} i \\ 1 \\ i \end{pmatrix} e^{(-1+2i)t} = \begin{pmatrix} i \\ 1 \\ i \end{pmatrix} e^t (\cos 2t + i \sin 2t)$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = C_1 e^{-t} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + C_2 e^t \begin{pmatrix} \sin t \\ \cos t \\ \sin t \end{pmatrix} + C_3 e^t \begin{pmatrix} \cos t \\ -\sin t \\ \cos t \end{pmatrix}$$

805

$$\begin{cases} \dot{x} = 2x - y - z \\ \dot{y} = 3x - 2y - z \\ \dot{z} = 2z - x + y \end{cases} \quad \lambda_1 = 0, \lambda_2 = \lambda_3 = 1$$

$$A = \begin{bmatrix} 2 & -1 & -1 \\ 3 & -2 & -1 \\ -1 & 1 & 2 \end{bmatrix}$$

$$A - \lambda_1 I = \begin{bmatrix} 2 & -1 & -1 \\ 3 & -2 & -1 \\ -1 & 1 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 \\ 2 & -1 & -1 \\ -1 & 1 & 2 \end{bmatrix}$$

$$e_1(-2, 0, 1) \quad e_2(-1, -1, 1)$$

$$\lambda = 1: A - \lambda I = \begin{bmatrix} 1 & -1 & -1 \\ 3 & -3 & -1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \text{ c.b.}$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = C_1 \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} + C_2 e^t \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + C_3 e^t \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

807

$$\begin{cases} \dot{x} = 5x - 4y - z \\ \dot{y} = 3x - 4y - 3z \\ \dot{z} = 2x - 4y \end{cases} \quad \lambda_1 = \lambda_2 = 2, \lambda_3 = -5$$

$$A = \begin{bmatrix} 5 & -4 & -1 \\ 3 & -4 & -3 \\ 2 & -4 & 0 \end{bmatrix}$$

$$\lambda_3 = -5: A - \lambda_3 I = \begin{bmatrix} 10 & -4 & -1 \\ 3 & 1 & -3 \\ 2 & -4 & 5 \end{bmatrix} \sim \begin{bmatrix} 2 & -4 & 5 \\ 1 & 5 & -9 \\ 0 & 14 & -21 \end{bmatrix}$$

$$e_1(1, 3, 2)$$

$$\lambda = 2: A - \lambda I = \begin{bmatrix} 3 & -4 & -1 \\ 3 & -6 & -3 \\ 2 & -4 & -2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow (2, 1, 0) \quad (1, 0, 1)$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} e^{-5t} + e^{2t} (C_2 \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + C_3 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix})$$

809

$$\begin{cases} \dot{x} = y - 2z - x \\ \dot{y} = x + y \\ \dot{z} = 2x + y - z \end{cases} \quad (\lambda_1 = 1, \lambda_2 = \lambda_3 = -1)$$

$$\lambda_1: A = \begin{bmatrix} -1 & 1 & -2 \\ 1 & 1 & 0 \\ 2 & 1 & -1 \end{bmatrix}$$

$$A - \lambda_1 I = \begin{bmatrix} -2 & 1 & -2 \\ 1 & 0 & 0 \\ 2 & 1 & -2 \end{bmatrix} \xrightarrow{+t, 0} C_1(0, 2, 1)$$

$$\lambda_{2,3}: A - \lambda_2 I = \begin{bmatrix} 0 & 1 & -2 \\ 1 & 2 & 0 \\ 2 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & -2 \\ 2 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{cases} x = (a + b + c) e^t \\ y = (c + b) e^t \\ z = (k + n) e^t \end{cases} \Rightarrow \begin{cases} a + b + c = -a - b + c + b - 2k \\ c + b = 4a + 4b + c + d \\ k + n = 2a + c - k, 2k + n = 2a + c \end{cases}$$

$$\begin{cases} a + b = -a - b + c + b - 2k \Rightarrow b = -2a \\ b = -b + d - 2n \Rightarrow d = 2n \\ c + b = 4a + c \Rightarrow d = 4a \\ d = 4b + d \Rightarrow b = 0 \\ k + n = 2a + c - k, 2k + n = 2a + c \Rightarrow n = 2b + d - n \Rightarrow d = 2n \end{cases}$$

$$n = \frac{1}{2}d, a = \frac{1}{4}b, b = -\frac{1}{2}d \quad c = 2n$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{1}{4}C_1 - \frac{1}{2}C_2 \\ C_2 + C_1 \\ C_2 + \frac{1}{2}C_1 \end{pmatrix} e^t + C_3 \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} e^t$$

813

$$\begin{cases} \dot{x}' = 2x - y \\ \dot{y} = x - 2y \end{cases}, \quad \begin{matrix} k = x \\ t = y \end{matrix} \Rightarrow \begin{cases} \dot{k} = 2x - y \\ \dot{t} = x - 2y \end{cases}$$

$$\Rightarrow A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & -1 & 0 & 0 \\ 1 & -2 & 0 & 0 \end{bmatrix} \Rightarrow \lambda = 1, -1, \pm i$$

5866

$$\dot{x} = Ax, \quad A = \begin{pmatrix} 2 & 0 & -1 \\ 1 & -1 & 0 \\ 3 & -1 & -1 \end{pmatrix}$$

$$A - \lambda I = \begin{vmatrix} 2-\lambda & 0 & -1 \\ 1 & -1-\lambda & 0 \\ 3 & -1 & -1-\lambda \end{vmatrix} =$$

$$= (2-\lambda)(-1-\lambda) - (-1+3+\lambda) = -\lambda^2 - 2\lambda + 2 = -\lambda^2 - 2\lambda + 2$$

$\lambda = 0$ up. 3

$$\begin{bmatrix} 2 & 0 & -1 \\ 1 & -1 & 0 \\ 3 & -1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 \\ 2 & 0 & -1 \\ 3 & -1 & -1 \end{bmatrix} \xrightarrow{R_2 - 2R_1, R_3 - 3R_1}$$

$\text{rg}(K) = 2 \Rightarrow 1 \text{ w.f.} + 2 \text{ up.}$

$$\Rightarrow x_1 = (a+bt)e^{2t} \\ x_2 = (c+dt)e^{2t} \\ x_3 = (k+nt)e^{2t}$$

$$\Rightarrow \begin{cases} 2a+2bt = 2a+2bt - knt \\ c+2dt = a+2bt - c-dt \\ k+2nt = 3a+2dt - c-dt - knt \end{cases}$$

$$\Rightarrow \begin{cases} a+b = 2a-k, \quad b = a-k, \quad k = a-b \\ b = 2b-n, \quad b = n, \quad n = b \\ c+d = a-c, \quad 2c+\frac{1}{2}b = a, \quad c = \frac{1}{2}a - \frac{1}{4}b \\ d = b-d, \quad b = 2d, \quad d = \frac{1}{2}b \\ k+n = 3a-c-k, \quad 2a+2b+n = 3a-c \\ n = 3b-d-n, \quad 2n = 3b-d \end{cases}$$

$$a = \frac{1}{2}a + b + \frac{1}{4}b = 0, \quad 4a - 2a + 7b + b = 0$$

$$b = 3b - \frac{1}{2}b - b = 0, \quad 2a + 5b = 0, \quad a = -\frac{5}{2}b$$

Kn. parat

5826

$$\begin{cases} \dot{x} = y + 2e^t \\ \dot{y} = x + t^2 \end{cases}, \quad \begin{cases} \dot{x} = y \\ \dot{y} = x \end{cases} \quad A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\Rightarrow \lambda^2 - 1 = 0, \quad \lambda = \pm 1$$

$$\lambda = 1: A - \lambda I = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t$$

$$\begin{cases} \dot{x} = y \\ \dot{y} = x + t^2 \end{cases} \xrightarrow{\lambda=0} \text{not separable}$$

$$x = at^2 + bt + c$$

$$y = kt^2 + lt + m$$

$$\dot{x} = 2at + b = kt^2 + lt + m$$

$$\dot{y} = (2k)t + l = x + t^2 = at^2 + bt + c + t^2$$

$$\begin{cases} 2at + b = kt^2 + lt + m \\ 2kt + l = at^2 + bt + c + t^2 \end{cases}$$

$$\begin{cases} k=0 \\ a=-1 \end{cases} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -t^2 - 2 \\ -2t \end{bmatrix}$$

$$\begin{cases} l=2a = -2 \\ b=2k = 0 \\ m=b = 0 \\ c=l = -2 \end{cases} \quad \text{by substitution if undetermined.}$$

$$2) \begin{cases} \dot{x} = y + 2e^t \\ \dot{y} = x \end{cases}$$

$$\begin{cases} x = (a+bt)e^t \\ y = (c+dt)e^t \end{cases} \quad \begin{cases} a+bt+b = c+dt+2 \\ c+dt+d = a+bt \end{cases}$$

$$\begin{cases} b=d \\ a+b = c+2 \\ c+d = a \end{cases} \quad d=b=1, \quad c=a-1$$

a - system ja 0 tu yuget b
2, 7c - a = 0.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} t \\ t-1 \end{bmatrix} e^t$$

5834

$$\begin{cases} \dot{x} = x + y \\ \dot{y} = x - 5y \end{cases}$$

$$\begin{cases} \dot{x} = x + y \\ \dot{y} = x \end{cases} \quad A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$|A - \lambda I| = \lambda^2 - \lambda - 2 \Rightarrow \lambda = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$1) \lambda = -1$$

$$(A + I)P = \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}, \Rightarrow c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$2) \lambda = 2$$

$$A - 2I = \begin{bmatrix} -1 & 1 \\ 1 & -2 \end{bmatrix} \Rightarrow c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t}$$

$$\begin{cases} \dot{x} = x + 2y \\ \dot{y} = x - 5e^{it} \end{cases} \quad \begin{cases} x = a e^{it} \\ y = b e^{it} \end{cases}$$

$$\begin{cases} a i = a + 2b \\ b i = a - 5 \end{cases} \quad a = \frac{2b}{i-1} = \frac{2b(1+i)}{(1+i)(i-1)} = \frac{2b(1+i)}{-1-1} = -b(1+i)$$

$$= -b(1+i),$$

$$b i + 5 = -b - b i$$

$$2b i + b = -5$$

$$b = -\frac{5}{2i+1} = -1+2i, \quad a = -(1+i)(2i+1) =$$

$$= (1+i)(1-2i) = 3-i$$

$$(3-i)e^{it} = \left(\begin{pmatrix} -3 \\ -1 \end{pmatrix} + i \begin{pmatrix} -1 \\ 2 \end{pmatrix} \right) (\cos t + i \sin t)$$

$$\begin{pmatrix} x \\ y \end{pmatrix}_2 = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \sin t + \begin{pmatrix} -1 \\ 2 \end{pmatrix} \cos t$$

§845

$$\begin{cases} \dot{x} = 2x - y \\ \dot{y} = 2y - x - 5e^{it} \sin t \end{cases}$$

$$1) \begin{cases} \dot{x} = 2x - y \\ \dot{y} = 2y - x \end{cases} \quad A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \quad \lambda_1 = 3, \lambda_2 = 1$$

$$\lambda = 3: \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} e_1(1, -1)$$

$$\lambda = 1: \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} e_2(1, 1)$$

$$\begin{pmatrix} x \\ y \end{pmatrix}_0 = c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t$$

$$\begin{cases} \dot{x} = 2x - y \\ \dot{y} = 2y - x - 5e^{t(1+i)} \end{cases} \quad \begin{cases} x = a e^{(1+i)t} \\ y = b e^{(1+i)t} \end{cases}$$

$$\begin{cases} a(1+i) = 2a - b \\ b(1+i) = 2b - a - 5 \end{cases}$$

$$\begin{cases} b = a(1-i) \\ a = \frac{-5(2i+1)}{-5} = 2i-1 \end{cases} \quad b = (1+i);$$

$$2) \left(\begin{pmatrix} -1 \\ 2 \end{pmatrix} + i \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right) (\cos t + i \sin t) e^t$$

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}_0 + e^t \left(\begin{pmatrix} 2 \\ 3 \end{pmatrix} \cos t + \begin{pmatrix} -1 \\ 1 \end{pmatrix} \sin t \right) e^t$$

§846

$$\begin{cases} \dot{x} = y + t g^2 t - 1 \\ \dot{y} = -x + t g t \end{cases}$$

$$1) A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \Rightarrow \lambda = \begin{bmatrix} +i \\ -i \end{bmatrix}$$

$$\lambda = i: A - \lambda I = \begin{bmatrix} -i & 1 \\ -1 & -i \end{bmatrix}, \quad e_1(1, i)$$

$$\begin{pmatrix} 1 \\ i \end{pmatrix} e^{it} = \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} + i \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right) (\cos t + i \sin t)$$

$$= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cos t + \begin{pmatrix} 0 \\ -1 \end{pmatrix} \sin t + i \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} \sin t + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cos t \right)$$

$$\begin{cases} x = c_1 \sin t + c_2 \cos t \\ y = c_1 \cos t - c_2 \sin t \end{cases}$$

$$\begin{cases} c_1 \sin t + c_2 \cos t = t g^2 t - 1 \\ c_1 \cos t - c_2 \sin t = t g t \end{cases} \quad \begin{matrix} \cdot \sin t \\ \cdot \cos t \end{matrix}$$

$$c_1 = \sin t (t g^2 t - 1) + \sin t = \frac{\sin^3 t}{\cos^2 t}$$

$$c_2 = t g t \cos t - \frac{1}{\cos t} = -\cos t$$

$$\Rightarrow c_2 = -\sin t + \bar{c}_2$$

$$c_1 = \frac{\sin^3 t}{\cos^2 t} \Rightarrow \int \frac{\sin^3 t}{\cos^2 t} d \cos t =$$

$$= \int \left(\frac{1}{\cos^2 t} - 1 \right) d \cos t = \cos t + \frac{1}{\cos t} + \bar{c}_1$$

$$x = \left(\cos t + \frac{1}{\cos t} + \bar{c}_1 \right) \sin t + (-\sin t + \bar{c}_2) \cos t =$$

$$= c_1 \sin t + c_2 \cos t + t g t$$

$$y = \left(\cos t + \frac{1}{\cos t} + \bar{c}_1 \right) \cos t - (-\sin t + \bar{c}_2) \sin t =$$

$$= c_1 \cos t - c_2 \sin t + \bar{c}$$

§848

$$\begin{cases} \dot{x} = 4x - 2y + \frac{2}{e^{t-1}} \\ \dot{y} = 6x + 3y - \frac{3}{e^{t-1}} \end{cases}$$

$$1) A = \begin{bmatrix} 4 & -2 \\ 6 & 3 \end{bmatrix} \Rightarrow (A - \lambda I) = \begin{bmatrix} 4-\lambda & -2 \\ 6 & 3-\lambda \end{bmatrix}$$

$$= 0 \Rightarrow (4-\lambda)(3-\lambda) - 12 = 0, \Rightarrow \lambda(4-\lambda) = 20$$

$$\lambda = 0: e_1(1, -2); \lambda = 1: e_2(2, -3)$$

$$\begin{cases} x = c_1 + 2c_2 e^{-t} \\ y = -2c_1 - 3c_2 e^{-t} \end{cases}$$

$$\begin{cases} c_1' + 2c_2' e^{-t} = \frac{2}{e^t - 1} \\ -2c_1' - 3c_2' e^t = \frac{-3}{e^t - 1} \end{cases}$$

$$\begin{aligned} c_1' &= 0 \\ c_2' &= \frac{e^t}{e^t - 1} \Rightarrow c_2 = \ln|e^t - 1| + c_2' \\ c_1 &= \end{aligned}$$

$$\begin{cases} x = c_1 + 2c_2 e^{-t} + 2e^{-t} \ln|e^t - 1| \\ y = -2c_1 - 3c_2 e^{-t} - 3e^{-t} \ln|e^t - 1| \end{cases}$$

Sp. p. m. a. n.

830

$$\begin{cases} \dot{x} = 4x + y - e^{2t} \\ \dot{y} = y - 2x \end{cases}$$

Seper: $A = \begin{bmatrix} 4 & 1 \\ -2 & 1 \end{bmatrix}$

$$(A - \lambda I) = (4 - \lambda)(1 - \lambda) + 2 = \lambda^2 - 5\lambda + 6$$

$$\lambda = 1, \lambda = 2$$

$$\lambda = 1: A - \lambda I = \begin{bmatrix} 3 & 1 \\ -2 & 0 \end{bmatrix} \Rightarrow v_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\lambda = 2: A - \lambda I = \begin{bmatrix} 2 & 1 \\ -2 & -1 \end{bmatrix} \Rightarrow v_2 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$y_1 = c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{2t} + c_2 \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{2t}$$

$$y_2 = \begin{cases} \dot{x} = 4x + y - e^{2t} \\ \dot{y} = y - 2x \end{cases} \quad \begin{aligned} x &= e^{2t} (c_1 + c_2 t) \\ y &= e^{2t} (c_1 + c_2 t) \end{aligned}$$

$$\Rightarrow \begin{cases} c_1 + 2c_2 = 4c_1 + c_2 + c_1 + c_2 - 1 \\ c_1 + 2c_2 = c_1 + c_2 - 2c_1 - 2c_2 \end{cases}$$

$$\Rightarrow \begin{cases} c_1 + 2c_2 = 4c_1 + c_2 + c_1 + c_2 - 1 \\ c_1 + 2c_2 = c_1 + c_2 - 2c_1 - 2c_2 \end{cases} \Rightarrow \begin{aligned} c_1 &= -1 \\ c_2 &= 1 \end{aligned}$$

$$\Rightarrow y_2: \begin{bmatrix} x \\ y \end{bmatrix} = e^{2t} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\Rightarrow y = y_1 + y_2$$

8X

$$\begin{cases} \dot{x} = 2x - y \\ \dot{y} = y - 2x + 18t \end{cases}$$

$$A = \begin{bmatrix} 2 & -1 \\ -2 & 1 \end{bmatrix} \quad |A - \lambda I| = (2 - \lambda)(1 - \lambda) - 2$$

$$\lambda^2 - 3\lambda = \lambda(\lambda - 3)$$

$$\lambda = 0: e, (1, 2)$$

$$\lambda = 3: A - 3I = \begin{bmatrix} -1 & -1 \\ -2 & -2 \end{bmatrix} \Rightarrow v_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\text{Seper. pen. } y = c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{3t}$$

$$y_2: \begin{cases} x = at + bt \\ y = ct + dt \end{cases}$$

$$\Rightarrow \begin{cases} b = 2a + 18t - c - dt \\ d = ct + dt - 2a - 2bt + 18t \end{cases}$$

$$\begin{cases} b = 2a + 18t - c - dt \\ c - 2a = 0 \\ d - 2b + 18 = 0, b = 9 \end{cases} \Rightarrow \begin{aligned} c &= 2a \\ d &= 2b - 18 \end{aligned}$$

$$\Rightarrow \begin{cases} x = at^2 + bt + c \\ y = dt^2 + et + f \end{cases} \Rightarrow \begin{aligned} 8a &= 2e - f \\ 2a &= 2b - c \\ 0 &= 2a - b \\ e &= f - 4c \\ 2d &= c - 2b + 17 \\ 0 &= d - 2a \end{aligned}$$

$$y_2 = \begin{pmatrix} 3t^2 + 2t \\ 6t^2 - 2t - 2 \end{pmatrix}$$

8X

$$\begin{cases} \dot{x} = 2x + y + 2e^t \\ \dot{y} = x + 2y - 3e^t \end{cases}$$

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \quad |A - \lambda I| = (2 - \lambda)^2 - 1$$

$$(2 - \lambda)(3 - \lambda)$$

$$1) y_1: c_1 e^t \begin{pmatrix} -1 \\ 1 \end{pmatrix} + c_2 e^{3t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$2) y_2: \begin{cases} \dot{x} = 2x + y + 2e^t \\ \dot{y} = x + 2y \end{cases} \quad \begin{aligned} x &= e^t (a + bt) \\ y &= e^t (c + dt) \end{aligned}$$

$$\Rightarrow \begin{cases} a+6b+b = 2a+2bt+c+bt+2 \\ c+4bt+d = a+bt+2c+2bt \end{cases}$$

$$\begin{cases} a+b = 2a+c+2, & a-d = 2a+c+2, \\ b = 2b+d, & b = -d \\ c+d = a+2c, & d = a+c \\ d = b+2b, & d = 3b \end{cases} \quad \begin{matrix} -c = 2a+c+2 \\ -a = -a-1 \end{matrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix}_1 = \begin{bmatrix} t \\ -1-t \end{bmatrix} e^t \quad \begin{matrix} b=1 \\ a=0 \\ c=-1 \end{matrix}$$

$$2) y_2: \begin{cases} \dot{x} = 2x+y \\ \dot{y} = x+4y-3e^{4t} \end{cases}$$

$$\Rightarrow \begin{cases} x = e^{4t}(a+bt) \\ y = e^{4t}(c+bt) \end{cases}$$

$$\begin{cases} 4a+4bt+b = 2a+2bt+c+bt \\ 4c+4bt+d = a+bt+2c+2bt-3 \end{cases}$$

$$\begin{cases} 4a+b = 2a+c \\ 4b = 2b+d, & 2b = d \\ 4c+d = a+2c-3 \\ 4d = b+2d, & b = 2d \end{cases}$$

$$\Rightarrow \begin{cases} x = e^{4t}(a_1+a_2t+a_3t^2) \\ y = e^{4t}(b_1+b_2t+b_3t^2) \end{cases}$$

$$\begin{cases} 4a_1+4a_2t+4a_3t^2+a_2+2a_3t = -2a_1+2a_2t+2a_3t^2 \\ 4b_1+4b_2t+4b_3t^2+b_2+2b_3t = a_1+a_2t+a_3t^2-3 \end{cases}$$

$$\Rightarrow \begin{cases} a_2 = -2a_1+b_1, & a_1 = \frac{1}{2}(-a_2) = -\frac{1}{2}b_1 \\ 2a_3 = -2a_2+b_2, & a_2 = \frac{1}{2}b_2 - a_3 = \frac{1}{2}b_2 - 2b_3 \\ 0 = -2a_3+b_3, & b_3 = 2a_3, a_3 = \frac{1}{2}b_3 \\ b_2 = a_1 - 2b_1 - 3, & a_1 = b_1 + 2b_3 + 3 \\ 2b_3 = a_2 - 2b_2, & a_2 = 2b_3 - 2b_2 \\ 0 = a_3 - 2b_3, & a_3 = 2b_3 \end{cases}$$

$$\Rightarrow \begin{cases} b_3 = 0 \\ a_3 = 0 \\ b_2 = 2 \\ b_1 = 0 \\ a_2 = 1 \\ a_1 = -\frac{1}{2} \end{cases} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix}_2 = \begin{bmatrix} -\frac{1}{2}+t \\ 2t \end{bmatrix} e^{4t}$$

$$y = y_0 + y_1 + y_2$$

844

$$\begin{cases} \dot{x} = 2y-x \\ \dot{y} = 4y-3x + \frac{e^{3t}}{e^{4t}+1} \end{cases}$$

$$A = \begin{bmatrix} -1 & 2 \\ -3 & 4 \end{bmatrix} \quad |A - \lambda I| = (-1-\lambda)(4-\lambda) + 6 = \lambda^2 - 3\lambda + 2 = (\lambda-1)(\lambda-2) = 0 \Rightarrow \lambda_1 = 1, \lambda_2 = 2 \Rightarrow \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$y_0 = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t}$$

$$\begin{cases} c_1 2e^t + c_2 e^t = 0 \\ c_1 4e^t + c_2 e^t = \frac{e^{3t}}{e^{4t}+1} \end{cases} \quad \begin{matrix} c_1 3e^t + c_2 e^t = \frac{e^{3t}}{e^{4t}+1} \end{matrix}$$

$$2c_1 e^t = \frac{e^{3t}}{e^{4t}+1} \Rightarrow c_1 = \frac{e^t}{2(e^{4t}+1)}$$

$$-c_2 e^t = \frac{e^{3t}}{e^{4t}+1}, \quad c_2 = -\frac{e^{2t}}{e^{4t}+1}$$

$$\Rightarrow c_1 = \frac{1}{2} \frac{de^{4t}}{e^{4t}+1} = \frac{1}{2} \ln(e^{4t}+1) + \tilde{c}_1$$

$$c_2 = -\frac{1}{2} \frac{d(e^{4t}+1)}{e^{4t}+1} = -\frac{1}{2} \ln(e^{4t}+1) + \tilde{c}_2$$

$$\Rightarrow \begin{cases} x = \frac{1}{2} \ln(e^{4t}+1) e^t + 2c_2 e^{2t} - e^t \ln(e^{4t}+1) + 3e^{2t} \ln(e^{4t}+1) \\ y = e^t e^{4t} + c_2 e^{4t} - e^t \ln(e^{4t}+1) + 3e^{2t} \ln(e^{4t}+1) \end{cases}$$

850

$$\begin{cases} \dot{x} = 3x - y \\ \dot{y} = 2x - y + 15e^{4t} \end{cases}$$

$$A = \begin{bmatrix} 3 & -1 \\ 2 & -1 \end{bmatrix}, \quad |A - \lambda I| = (3-\lambda)(-1-\lambda) + 4 = \lambda^2 + \lambda - 3\lambda + 1 = \lambda^2 - 2\lambda = \lambda(\lambda-2) = 0 \Rightarrow \lambda_1 = 0, \lambda_2 = 2$$

$$\lambda = 1: |A - \lambda I| = \begin{bmatrix} 2 & -1 \\ 2 & -2 \end{bmatrix}$$

$$\begin{cases} x = (a+bt)e^t \\ y = (c+dt)e^t \end{cases} \Rightarrow \begin{cases} a+bt+b = 2a+2bt-2c-4t \\ c+dt+d = 2a+2bt-c-4t \end{cases}$$

$$\begin{cases} b = 2a-2c, & 2a = b+2c \\ 0 = 2bt-2d, & b = d \\ 2c+d = 2a, & d = 2b-d \Rightarrow d = 2b \Rightarrow 2b = 2b \end{cases}$$

$$\Rightarrow x = (\frac{1}{2}c_1 + c_2 + c_3 t)e^{4t} \\ y = (c_2 + c_3 t)e^{4t}$$

$$y_0 = \begin{pmatrix} \frac{1}{2}c_1 + c_2 + c_3 t \\ c_2 + c_3 t \end{pmatrix} e^t$$

$$\begin{cases} c_1(\frac{1}{2}+t)e^t + c_2 e^t = 0 \\ c_1 t e^t + c_2 e^t = 15e^{4t} \end{cases}$$

$$\Rightarrow c_1 = \frac{1}{2}b_1 + c_2, \quad b = d, \quad c_1$$

$$\frac{1}{2} \dot{c}_1 = 15t^2 e^t, \quad c_1 = -20t^2 + c_1$$

$$\Rightarrow c_1 = -20 \cdot \frac{2}{3} t^{\frac{3}{2}} + c_1 = -20t^{\frac{3}{2}} + c_1$$

$$c_2 = 15t^2 - c_1 t = 15t^2 + 20t^{\frac{3}{2}}$$

$$\Rightarrow c_2 = 15 \cdot \frac{2}{3} t^{\frac{3}{2}} + 20 \cdot \frac{2}{3} t^{\frac{3}{2}} + c_2 = 10t^{\frac{3}{2}} + 12t^{\frac{3}{2}} + c_2$$

$$\Rightarrow \begin{cases} x = (15t^2 - 20t^{\frac{3}{2}} + c_1)(\frac{1}{2} + t)e^t + (10t^{\frac{3}{2}} + 12t^{\frac{3}{2}} + c_2)e^t \\ y = (10t^{\frac{3}{2}} + 12t^{\frac{3}{2}} + c_2 + 20t^{\frac{3}{2}} + c_1 t)e^t \end{cases}$$

Класс.

Зависимость от параметров.

Задача

$$y'(x) = y(x) + \mu(x+y^2)$$

$$y(0) = 1$$

$$\frac{\partial y}{\partial \mu} \Big|_{\mu=0} = ?$$

$$y(x, \mu) = y_0(x) + \mu y_1(x) + \mu^2 y_2(x) + \dots$$

$$\frac{\partial y}{\partial \mu} \Big|_{\mu=0} = y_1(x), \quad y_0'(x) + \mu y_1'(x) + \mu^2 y_2'(x) + \dots = y_0(x) + \mu y_1(x) + \mu^2 y_2(x) + \dots$$

$$\Rightarrow y_0(x) + \mu y_1(x) + \mu^2 y_2(x) + \dots + \mu x + \mu(y_0(x) + \mu y_1(x) + \dots)^2$$

$$\mu y_1(x) + \mu^2 y_2(x) + \dots + \mu x + \mu(y_0(x) + \mu y_1(x) + \dots)^2$$

$$\text{при } \mu^0 \quad 1) \begin{cases} y_0'(x) = y_0(x) \\ y_0(0) = 1 \end{cases} \quad y_0(x) = e^x$$

$$\text{при } \mu^1 \quad 2) \begin{cases} y_1'(x) = y_1(x) + x + e^{2x} \\ y_1(0) = 0 \end{cases}$$

$$\Rightarrow \text{сгн} - : y_1(x) = ce^x$$

$$y_{12} = ax + b \Rightarrow \begin{cases} a = ax + b + x \\ a = -1, b = -1 \end{cases}$$

$$\Rightarrow y_{12} = -x - 1$$

$$y_{12} = ae^{2x} \Rightarrow 2a = a + 1, a = 1$$

$$y_{12} = e^{2x}$$

$$y_1(x) = ce^x - x - 1 + e^{2x}$$

$$y_1(0) = 0 \Rightarrow c = 0$$

$$2) \frac{\partial y}{\partial \mu} \Big|_{\mu=0} \equiv$$

$$\textcircled{2} e^{2x} - x - 1.$$

Задача

$$\begin{cases} \frac{dx}{dt} = \frac{x}{t} + \mu t e^{-x} \\ x(0) = 1 \end{cases} \quad \frac{dx}{dt} \Big|_{\mu=0} = ?$$

$$x(t, \mu) = x_0(t) + \mu x_1(t) + \dots$$

$$x_0'(t) + \mu x_1'(t) + \dots = \frac{x_0(t)}{t} + \mu \frac{x_1(t)}{t} + \dots + \mu t e^{-x}$$

$$e^{-x_0(t) - \mu x_1(t) - \dots} = e^{-x_0(t)} \cdot e^{-\mu x_1(t) - \dots}$$

$$x_0(1) + \mu x_1(1) + \dots = 1$$

$$1) \begin{cases} x_0'(t) = \frac{x_0(t)}{t} \\ x_0(1) = 1 \end{cases} \Rightarrow x_0 = c t^2$$

$$c = 1 \Rightarrow x_0 = t^2$$

$$2) \begin{cases} x_1'(t) = \frac{x_1(t)}{t} + t e^{-t^2} \\ x_1(1) = 0 \end{cases} \quad (\text{exp разложим в ряд})$$

$$x_1(t) = ct, \quad x_1'(t) = c t + c$$

$$\Rightarrow c = e^{-t^2} + c \quad x_1(t) = ct - e^{-t^2} t$$

$$x_1(1) = c - \frac{1}{e} = 0 \Rightarrow c = \frac{1}{e}$$

$$\Rightarrow x_1(t) = \frac{\partial x}{\partial \mu} \Big|_{\mu=0} = \frac{1}{e} t - e^{-t^2} t$$

Задача

$$\begin{cases} \dot{x} = 4t y^2 \\ \dot{y} = 1 + 5\mu x \\ x(0) = 0 \\ y(0) = 0 \end{cases} \quad \frac{\partial x}{\partial \mu} \Big|_{\mu=0} = ? \quad x_1(t)$$

$$x(t, \mu) = x_0(t) + \mu x_1(t) + \dots$$

$$y(t, \mu) = y_0(t) + \mu y_1(t) + \dots$$

Разложим в ряд:

$$x_0'(t) + \mu x_1'(t) + \mu^2 x_2'(t) + \dots = 4t(y_0(t) + \mu y_1(t) + \dots)^2$$

$$y_0'(t) + \mu y_1'(t) + \mu^2 y_2'(t) + \dots = 1 + 5\mu(x_0(t) + \mu x_1(t) + \dots)$$

$$x_0'(t) = 4t y_0^2(t)$$

$$y_0'(t) = 1$$

$$x_0(0) = 0$$

$$y_0(0) = 0$$

$$y = t + c, \quad c = 0 \Rightarrow y = t$$

$$x_0 = t^2 + c, \quad x_0(0) = 0 \Rightarrow c = 0$$

$$2) \begin{cases} x_1'(t) = 4t \cdot 2y_0(t) \cdot y_1(t) = 8t^2 y_1(t) \\ y_1'(t) = 5x_0(t) = 5t^2 \\ x_1(0) = 0 \\ y_1(0) = 0 \end{cases}$$

$$y_1 = t^5 + c, \quad y_1(0) = 0 \Rightarrow c = 0$$

$$x_1 = \int 8t^7 dt = t^8$$

1070

$$\dot{x} = xy + t^2 \quad u = \frac{2x}{2y}, \quad \delta = \frac{2y}{2y}$$

$$\begin{cases} \dot{y} = -y^2 \\ x(1) = x_0 \\ y(1) = y_0 \end{cases} \quad \begin{cases} \dot{u} = uy + \delta x \\ 2\dot{v} = 2y\delta \\ u(1) = 0 \\ v(1) = 1 \end{cases}$$

$$\frac{2x}{2y} \Big|_{x=3} = \frac{2x}{2y} \quad \frac{2x}{2y} = \frac{2x}{2y}$$

$$\frac{1}{y} = -\frac{1}{y^2} \quad \frac{1}{y} = -\frac{1}{y^2}$$

$$\frac{1}{y} = \frac{t}{2} + C, C = 0 \quad y = \frac{2}{t}$$

$$\dot{u} = \frac{2x}{t} + (2t^2 + t^3) \delta \quad u = ?$$

$$2\dot{v} = -\frac{4}{t} \delta \quad \Rightarrow \delta = \frac{C}{t^2}, C = 1, \delta(t) = \frac{1}{t^2}$$

$$u(1) = 0 \quad u_{\text{gen}} = Ct^2 \Rightarrow$$

$$2) C't^2 + 2Ct = 2Ct + 2t \quad C' = \frac{2}{t^2} + \frac{1}{t}, C = -\frac{2}{t} + \ln t + \tilde{C}$$

$$\Rightarrow u = -2t + t^2 \ln t + \tilde{C}t^2, 0 = -2 + \tilde{C} \Rightarrow \tilde{C} = 2$$

$$\frac{2x}{2y} \Big|_{x=3} = 2t^2 + t^2 \ln t - 2t$$

$$y_0 = 2$$

1074

$$y'(x) = 4\mu x - y^2, y(1) = 1$$

$$y(x, \mu) = y_0(x) + \mu y_1(x) + \dots$$

$$y_0(1) + \mu y_1(1) + \dots = 1$$

$$1) \begin{cases} y_0'(x) = -y_0^2 \\ y_0(1) = 1 \end{cases} \quad y_0 = \frac{1}{x+C} \Rightarrow y_0(x) = \frac{1}{x+C}$$

$$2) \begin{cases} y_1'(x) = 4x - 2y_0 y_1 \\ y_1(1) = 0 \end{cases} \Rightarrow y_1(x) = Cx^2$$

$$y_2'(x) = -y_1^2(x) - 2y_0(x)y_1(x) = -(x^2+x^2)^2 - \frac{2}{x}y_1(x) =$$

$$y_2(1) = 0 \Rightarrow y_2(x) = Cx^2 \Rightarrow y_2(x) = -\frac{x^5}{7} + x^{-3} + \frac{2x}{3} - \frac{32}{21}x^2$$

1076

$$xy' = \mu x^2 + \ln y$$

$$y = y_0 + \mu y_1 + \mu^2 y_2 + \dots$$

$$x(y_0 + \mu y_1 + \dots) = \mu x^2 + \ln(y_0 + \dots)$$

$$y_0(1) + \dots = 1 \quad f = \ln(y_0 + \dots)$$

$$f(y, \mu) = f|_{\mu=0} + \frac{\partial f}{\partial \mu} \Big|_{\mu=0} \mu + \frac{\partial^2 f}{\partial \mu^2} \Big|_{\mu=0} \frac{\mu^2}{2} + \dots$$

$$f|_{\mu=0} = \ln(y_0), \frac{\partial f}{\partial \mu} = \frac{1}{y_0 + \mu y_1 + \dots} (y_1 + 2y_2 + \dots)$$

$$\frac{\partial^2 f}{\partial \mu^2} = \frac{(2y_2 + \dots)(y_0 + \dots) - (y_1 + \dots)(y_1 + \dots)}{(y_0 + \dots)^2}$$

$$\Rightarrow \frac{\partial^2 f}{\partial \mu^2} \Big|_{\mu=0} = \frac{2y_2 y_0 - y_1^2}{y_0^2}$$

$$1) xy_0' = \ln y_0 \Rightarrow y_0(x) = 1 \quad x((\ln y_0)') = \ln y_0$$

$$xy_1' = x^2 + y_1 \Rightarrow y_1 = Cx$$

$$y_1(1) = 0 \Rightarrow y_1 = x^2 - x$$

$$b) xy_2' = \frac{2xy_0 - y_1^2}{2y_0^2} = y_2 - \frac{x^4}{2} + x^3 - \frac{x^2}{2}$$

$$y_2(1) = 0 \quad y_2' = \frac{y_1}{x}, y_2 = Cx$$

$$y_2' = Cx + C \quad C = -\frac{x^4}{6} + \frac{x^3}{2} - \frac{x}{2} + \tilde{C}$$

$$\Rightarrow y_2 = -\frac{x^4}{6} + \frac{x^3}{2} - \frac{x^2}{2} + \frac{x}{6}$$

Dip

1065

$$y' = 2x + \mu y^2 \quad \frac{\partial y}{\partial \mu} \Big|_{\mu=0}$$

$$y(x, \mu) = y_0(x) + \mu y_1(x) + \mu^2 y_2(x) + \dots$$

$$\begin{cases} y_0' + \mu y_1' + \dots = 2x + \mu(y_0 + \mu y_1 + \dots)^2 \\ y_0(0) + \mu y_1(0) + \dots = \mu - 1 \end{cases}$$

$$\frac{\partial y}{\partial \mu} \Big|_{\mu=0} = g(x)$$

$$1) \begin{cases} y_0' = 2x \Rightarrow y_0 = x^2 + C \\ y_0(0) = -1; y_1(0) = C = -1 \Rightarrow \\ \Rightarrow y_0 = x^2 - 1 \end{cases}$$

$$2) \begin{cases} y_1' = y_0^2 \Rightarrow y_1 = \int (x^2 - 1)^2 dx = \int (x^4 - 2x^2 + 1) dx = \frac{x^5}{5} - \frac{2x^3}{3} + x + C \\ y_1(0) = C = 1 \end{cases}$$

$$g(x) = \frac{x^5}{5} - \frac{2x^3}{3} + x + 1$$

Σ 1066

$$\begin{cases} y' = y + y^2 + xy^3 \\ y(2) = y_0 \end{cases} \quad \frac{\partial y}{\partial y_0} \Big|_{y_0=0} = ? \quad \left| \frac{\partial y}{\partial y_0} = u(x, y) \right|_{y_0=0} = u(x, y_0)$$

$$y(x, \mu) = y_0(0) + y_1 \mu + \dots$$

$$\begin{cases} y_0' + \mu y_1' + \dots = y_0 + y_1 \mu + \dots + (y_0 + y_1 \mu + \dots)^2 + x(y_0 + y_1 \mu + \dots)^3 \\ y(2) = y_0(2) + y_1(2) \mu + \dots = \mu \end{cases}$$

$$1) \begin{cases} y_0' = y_0 + y_0^2 + x y_0^3 = y_0(y_0 + y_0 + x) \\ y_0(2) = 0 \end{cases}$$

$$\begin{cases} \frac{\partial u}{\partial x} = 1 + 2y(x, y_0) \cdot u + 3xy^2 u \\ u(2, 0) = 1, u(x, y_0) = \frac{\partial y}{\partial y_0} \end{cases}$$

$$\text{Für } y_0 = 0 \Rightarrow x \rightarrow \frac{\partial y}{\partial y_0} \Big|_{y_0=0}$$

$$\begin{cases} \frac{\partial u(x, 0)}{\partial x} = u(x, 0) + 2y(x, 0)u(x, 0) + 3xy^2(x, 0)u(x, 0) \\ u(2, 0) = 1 \end{cases}$$

$$y(x, 0) = 0 \Rightarrow y'(x, 0) = y_0'(x, 0) + y_1'(x, 0) + x y_0^3(x, 0)$$

$$y_1(2, 0) = 0 \Rightarrow \frac{\partial u(x, 0)}{\partial x} = u(x, 0), u(2, 0) = 1$$

$$\Rightarrow u(x, 0) = e^{x-2} \Rightarrow \frac{\partial y}{\partial y_0} \Big|_{y_0=0} = e^{x-2}$$

Σ 671

$$\begin{cases} \dot{x} = x + y & x(0) = 1, y(0) = -2 \\ \dot{y} = 2x + \mu y^2 & \frac{\partial y}{\partial \mu} \Big|_{\mu=0} = ? \end{cases}$$

$$x(t, \mu) = x_0 + \mu x_1 + \dots$$

$$y(t, \mu) = y_0 + \mu y_1 + \dots$$

$$\begin{cases} x_0' + \mu x_1' + \dots = x_0 + \mu x_1 + \dots + y_0 + \mu y_1 + \dots \\ y_0' + \mu y_1' + \dots = 2x_0 + \mu(2x_1 + \dots) + \mu(y_0 + \mu y_1 + \dots)^2 \\ x_0(0) + \mu x_1(0) + \dots = 1 + \mu \\ y_0(0) + \mu y_1(0) + \dots = -2 \end{cases}$$

$$1) \begin{cases} x_0' = x_0 + y_0 \\ y_0' = 2x_0 + y_0^2 \Rightarrow y_0 = 2 \cdot \int x_0 dx \\ x_0(0) = 1 \\ y_0(0) = -2 \end{cases} \Rightarrow \int x_0 dx = x_0 + 2 \int x_0 dx \Rightarrow x_0 = \frac{2}{x_0} \Rightarrow \frac{dx_0}{x_0} = -dx_0$$

$$\Rightarrow \frac{dx_0}{x_0} = -dx_0 \Rightarrow \ln|x_0| = -x_0 + C \Rightarrow x_0 = e^{-x_0} \Rightarrow x_0(0) = C = 1 \Rightarrow x_0 = e^{-x_0}$$

$$\Rightarrow y_0 = -2e^{-x_0}$$

$$2) \begin{cases} x_1' = x_1 + y_1 \\ y_1' = 2x_1 + y_0^2 = 2x_1 + 4e^{-2x_0} \\ x_1(0) = 1 \\ y_1(0) = 0 \end{cases} \Rightarrow y_1 = 2 \int x_1 dx + 4 \int e^{-2x_0} dx$$

$$\Rightarrow x_1' = -x_1 + 2e^{-2x_0} + C_1$$

$$u(t, \mu) = \frac{\partial x(t, \mu)}{\partial \mu}, v(t, \mu) = \frac{\partial y(t, \mu)}{\partial \mu}$$

$$\frac{du(t, 0)}{dt} = u(t, 0) + v(t, 0), u(0, 0) = 1$$

$$\frac{dv(t, 0)}{dt} = 2u(t, 0) + y^2(t, 0), v(0, 0) = 0$$

$$\begin{cases} u'(t, 0) = u(t, 0) + v(t, 0) \\ v'(t, 0) = 2u(t, 0) + y^2(t, 0) \end{cases} \Rightarrow \begin{cases} u(0, 0) = 1 \\ v(0, 0) = 0 \end{cases}$$

$$y(t, 0) \text{ nach } u, v: \begin{cases} \dot{x}(t, 0) = x(t, 0) + y(t, 0) \\ y(t, 0) = 2x(t, 0) \end{cases} \Rightarrow \begin{cases} \dot{x}(t, 0) = x(t, 0) + 2x(t, 0) \\ \dot{y}(t, 0) = 2x(t, 0) + y(t, 0)^2 \end{cases}$$

$$x(t, 0) = \frac{1}{2} y(t, 0) \Rightarrow \ddot{y}(t, 0) - \dot{y}(t, 0) - 2y(t, 0) = 0$$

$$y(0, 0) = -2, \dot{y}(0, 0) = 2$$

$$\Rightarrow y(t, 0) = -2e^{-t}$$

$$u_y(x) \Rightarrow \begin{cases} \ddot{v}(t, 0) - \dot{v}(t, 0) - 2v(t, 0) = -12e^{-2t} \\ v(0, 0) = 0 \\ \dot{v}(0, 0) = 6 \end{cases}$$

$$\Rightarrow v(t, 0) = 2e^{-t} + e^{-2t} - 3e^{-4t}$$

Σ 1072

$$\begin{cases} \ddot{x} - \dot{x} = (x+1)^2 + \mu x^2 \\ x(0) = \frac{1}{2} \\ \dot{x}(0) = -1 \end{cases} \quad \frac{\partial x}{\partial \mu} \Big|_{\mu=0} = ?$$

$$u(t, \mu) = \frac{2x(t, \mu)}{\mu}; \Rightarrow u'(t, 1) - 2u'(t, 1) - 2u = -x^2(t, 1), u(0, 1) = 0, u'(0, 1) = 0.$$

$$\Rightarrow \begin{cases} x'(t, 1) - x'(t, 1) = 2x(t, 1) + 1 \\ x(0, 1) = \frac{1}{2} \\ \dot{x}(0, 1) = -1 \end{cases}$$

$$\Rightarrow x(t, 1) = e^{-t} - \frac{1}{2}; \ddot{u}(t, 1) - \dot{u}(t, 1) - 2u(t, 1) = -(e^{-t} - \frac{1}{2})^2; u(0, 1) = 0, u'(0, 1) = 0$$

$$\Rightarrow u(t, 1) = \frac{2x}{\mu} \Big|_{\mu=1} = \frac{1}{8} + e^t \left(\frac{5}{26} - \frac{1}{3} \right) - \frac{1}{4} e^{-2t} - \frac{1}{72} e^{2t}$$

1075

$$\begin{cases} y' = \frac{2}{y} - 5x \\ y(0) = 2 \end{cases} \quad 2, 3 \text{ змен. погр.}$$

$$y = y_0 + \mu y_1 + \dots \Rightarrow \begin{cases} y_0' + \mu y_1' + \dots = \frac{2}{y_0 + \mu y_1 + \dots} - 5x \\ y_0(0) + \mu y_1(0) + \dots = 2 \end{cases}$$

$$\frac{2}{y_0 + \mu y_1 + \dots} = \frac{2}{y_0} \left(1 - \mu \frac{y_1}{y_0} + \mu^2 \frac{y_1^2}{y_0^2} - \dots \right) = \frac{1}{y_0} \left(1 - \mu \frac{y_1}{y_0} + \mu^2 \frac{y_1^2}{y_0^2} - \dots \right)$$

$$\Rightarrow \begin{cases} y_0' + \mu y_1' + \dots = \frac{2}{y_0} - \mu \frac{2y_1}{y_0^2} + \mu^2 \frac{2y_1^2}{y_0^3} - \dots - 5x \\ y_0(0) + \mu y_1(0) + \dots = 2 \end{cases}$$

$$\Rightarrow 1) \begin{cases} y_0' = \frac{2}{y_0} \\ y_0(0) = 2 \end{cases} \Rightarrow y_0 = 2x + C \Rightarrow 2 = 2 + C \Rightarrow C = 0 \Rightarrow y_0 = 2x$$

$$2) \begin{cases} \mu y_1' = -\frac{2y_1}{y_0} - 5x \\ y_1(0) = 0 \end{cases}, y_1' + \frac{2}{y_0} y_1 = -5x$$

$$1) y_1 = ax^2 \Rightarrow y_1' = 2ax \Rightarrow 2ax + \frac{2}{2x} ax = -5x \Rightarrow 2a + \frac{1}{2}a = -5 \Rightarrow 4a + a = -10 \Rightarrow a = -2$$

$$2) \frac{dy_1}{y_1} = -\frac{2}{y_0} dx$$

$$3) \begin{cases} y_2' = \frac{2}{y_0} \left(-\frac{y_1}{y_0} + \frac{y_1^2}{y_0^2} \right) \\ y_2(0) = 0 \end{cases} \Rightarrow \ln|y_2| = \frac{2}{y_0} \ln|x| + C \Rightarrow y_2 = x^{-\frac{1}{2}} \cdot C + 2x^2$$

$$y_2' = \frac{2}{y_0^3} (y_1^2 - y_0^2) = \frac{1}{4} x^{-\frac{1}{2}} (4(x^2 - x^2) - y_2^2)$$

$$\Rightarrow y_2 = \frac{1}{4} x^{-\frac{1}{2}} y_2 = x^{-\frac{1}{2}} \left(x^{-\frac{1}{2}} - x^{\frac{1}{2}} \right)^2$$

$$\Rightarrow y_2 = \frac{1}{4} x^{-\frac{1}{2}} \left(x^{-\frac{1}{2}} - x^{\frac{1}{2}} \right)^2 = \frac{1}{4} x^{-\frac{1}{2}} \left(x^{-1} - 2x^0 + x^1 \right) = \frac{1}{4} x^{-\frac{1}{2}} (-2 + x^{\frac{1}{2}})$$

$$\odot y_1' = e^{\frac{1}{2}x} \cdot \frac{1}{2} \cdot (-\frac{1}{2}) x^{-\frac{1}{2}} + C e^{\frac{1}{2}x} \Rightarrow -\frac{1}{4} x^{\frac{3}{2}} C e^{\frac{1}{2}x} + C e^{\frac{1}{2}x} + \frac{1}{4} x^{-\frac{3}{2}} C e^{\frac{1}{2}x} = x^{-\frac{1}{2}} + 2 + x^{\frac{1}{2}} \Rightarrow$$

$$y_2 = \frac{1}{4} x^{\frac{3}{2}} - \frac{1}{2} x + \frac{25}{12} x^{-\frac{1}{2}} - x^{-\frac{3}{2}}$$

1078

$$\begin{cases} y' = e^{y-x} + \mu y \\ y(0) = -\mu \end{cases}$$

$$y(x, \mu) = y_0 + \mu y_1 + \dots$$

$$\Rightarrow y_0' + \mu y_1' + \dots = e^{y_0-x+\mu y_1+\dots} + \mu y_1 + \dots = e^{y_0-x} \left(1 + \mu \frac{y_1}{y_0} + \mu^2 \frac{y_1^2}{y_0^2} + \dots \right) + \mu y_1 + \dots$$

$$\Rightarrow 1) y_0' = e^{y_0-x} \Rightarrow e^{y_0-x} y_0' = e^{y_0-x} \Rightarrow y_0 = \int e^{-x} dx \Rightarrow y_0(0) = 0 \Rightarrow e^{-y_0} = e^{-x} + C \Rightarrow 1 = 1 + C \Rightarrow C = 0 \Rightarrow y_0 = -x$$

$$2) 1) \begin{cases} y_1' = e^{y_0-x} \\ y_1(0) = 0 \end{cases} \Rightarrow y_1 = \int e^{-x} dx \Rightarrow y_1(0) = 0 \Rightarrow y_1 = -x$$

$$\Rightarrow y_1 = -x$$

$$\Rightarrow y_1(x) = -(x); \Rightarrow y_2(x) = \frac{1}{2} (e^x - x^2 - 2x - 1)$$

1081

1081

$$\begin{cases} y' = y^2 - x \\ y(0) = 1 \end{cases}$$

Можно использовать метод разложения в ряд по степеням x .

$$y(x) = \sum_{n=0}^{\infty} a_n x^n \Rightarrow a_1 + 2a_2 x + \dots = (a_0 + a_1 x + \dots)^2 - x$$

$$\Rightarrow \begin{cases} a_1 = a_0^2 \\ 2a_2 = 2a_0 a_1 - 1 \\ 3a_3 = a_1^2 + 2a_0 a_2 \\ 4a_4 = 2a_1 a_2 + 2a_0 a_3 \end{cases} \Rightarrow \begin{cases} a_0 = 1 \\ a_1 = 1 \\ a_2 = \frac{1}{2} \\ a_3 = \frac{2}{3} \\ a_4 = \frac{7}{12} \end{cases} \Rightarrow y(x) = 1 + x + \frac{1}{2} x^2 + \frac{2}{3} x^3 + \frac{7}{12} x^4 + \dots$$

1084

$$\begin{cases} y' = 2x + a_3 y = 2x + 1 - \frac{y^2}{2} + \frac{y^4}{4} + \dots \\ y(0) = 0 \end{cases}$$

$$\begin{cases} a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \dots = 0 \\ 2x + 1 - \frac{(a_0 + a_1 x + \dots)^2}{2} + \frac{(a_0 + a_1 x + \dots)^4}{4} + \dots \\ a_0 = 0 \end{cases}$$

$$\begin{cases} a_0 = 0 \\ a_1 = 1 \\ 2a_2 = 2 \Rightarrow a_2 = 1 \\ 3a_3 = \frac{1}{2} - \frac{a_1^2}{2} = -\frac{1}{2} \Rightarrow a_3 = -\frac{1}{6} \\ 4a_4 = -1 \Rightarrow a_4 = -\frac{1}{4} \end{cases}$$

$$y(x) = x + x^2 - \frac{1}{6}x^3 - \frac{1}{4}x^4 + \dots$$

Нахождение и.и.г. рен.
до буге степен. ряда

S1102

$$(1-x^2)y'' - 4xy' = 2y' = 0$$

$$y = \sum_{n=0}^{\infty} a_n x^n \Rightarrow \sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} - 4x \sum_{n=1}^{\infty} n a_n x^{n-1} - 2 \sum_{n=1}^{\infty} a_n x^n = 0$$

$$\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} - \sum_{n=1}^{\infty} 4n a_n x^n - 2 \sum_{n=1}^{\infty} a_n x^n = 0$$

$$\begin{cases} 2a_2 - 2a_0 = 0, a_0 = a_2 \\ 6a_3 - 4a_1 - 2a_1 = 0 \Rightarrow a_3 = a_1 \\ (n+2)(n+1)a_{n+2} - (4n+2)a_n = 0 \end{cases}$$

$$\Rightarrow a_{n+2} = a_n, \forall n, n \geq 2$$

$$a_0 = 1 \Rightarrow y_1(x) = 1 + x^2 + x^4 + x^6 + \dots$$

$$a_0 = 0 \Rightarrow y_2(x) = x + x^3 + x^5 + \dots$$

S1104

$$(1-x)y'' - 2y' + y = 0$$

$$y = \sum_{n=0}^{\infty} a_n x^n \Rightarrow (1-x) \sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} - 2 \sum_{n=1}^{\infty} n a_n x^{n-1} + \sum_{n=0}^{\infty} a_n x^n = 0$$

$$2a_2 - 2a_1 + a_0 = 0$$

$$\Rightarrow (n+2)(n+1)a_{n+2} - n(n+1)a_{n+1} - 2(n+1)a_{n+1} + a_n = 0$$

$$\Rightarrow a_0 = 0, a_1 = 1 \Rightarrow 6a_3 - 6a_2 + a_1 = 0 \Rightarrow a_3 = -\frac{1}{2}$$

$$(x) a_0 = 1, a_1 = 0 \Rightarrow 12a_4 + \dots = 0 \Rightarrow a_4 = -\frac{1}{24}$$

$$\Rightarrow y_1(x) = \dots$$

$$y_2(x) = \dots$$

Представл. в решении
до буге степен. ряда

S11010

$$xy'' + 2y' + xy = 0$$

$$\text{Общ. рен. } y(x) = x^2 \sum_{n=0}^{\infty} a_n x^n$$

$$\Rightarrow 2(2+1) + 22 = 0; 2^2 + 2 = 0 \Rightarrow 2 = 0$$

$$f = -1 \Rightarrow y(x) = \sum_{n=0}^{\infty} a_n x^{n-1} = \frac{a_0}{x} + a_1 + a_2 x + \dots$$

$$y'(x) = -\frac{a_0}{x^2} + a_2 + 2a_3 x + \dots$$

$$\Rightarrow \frac{2a_0}{x^2} + 2a_3 x + 6a_4 x^2 + \dots - \frac{2a_0}{x^2} + 2a_2 + 4a_3 x + \dots$$

$$\begin{cases} 2a_2 = 2a_0 \\ 2a_3 + 4a_1 = 0 \\ 6a_4 + 6a_1 + a_2 = 0 \end{cases} \Rightarrow \begin{cases} a_0 = 1 \\ a_1 = 0 \\ a_2 = 1 \\ a_3 = -\frac{1}{2} \\ a_4 = -\frac{1}{24} \end{cases}$$

$\Rightarrow \begin{cases} a_0 = 1 \\ a_1 = 0 \end{cases}$ т.к. сумма степен. рядов

Эп

§10.3

$$\begin{cases} y' = y + x e^x \\ y(0) = 0 \end{cases}$$

$$y = \sum_{k=0}^{\infty} a_k x^k \Rightarrow y' = a_1 + 2a_2 x + \dots = e^x \cdot (1 + a_1 + \dots)$$

$$\begin{aligned} y(0) &= 0 \cdot 0 \cdot e^0 = 0 \\ y'(0) &= y(0) + e^0 + x \cdot e^x \cdot y(0)|_0 = 1 \\ y''(0) &= 1 \\ y'''(0) &= 4 \end{aligned}$$

$$y(x) = 0 + 0 \cdot \frac{x}{1!} + 1 \cdot \frac{x^2}{2!} + 1 \cdot \frac{x^3}{3!} + \frac{4 \cdot x^4}{4!}$$

§11.3

$$x^2 y'' - x y' + (x-2)y = 0$$

$$x^2 \sum_{n=0}^{\infty} a_n x^n - x \sum_{n=0}^{\infty} (n+2)(n+1) a_n x^n + (x-2) \sum_{n=0}^{\infty} a_n x^n = 0$$

$$-2a_0 + (2+2)(2-1)a_0 = 0 \quad | a_0 \neq 0$$

$$\Rightarrow 2(2-1) = 2, \quad 2^2 - 2 + 2 = 0$$

$$z = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$z = 2: x^2 \sum_{n=0}^{\infty} a_n x^n$$

$$\Rightarrow \sum_{n=0}^{\infty} (n+2)(n+1) a_n x^n - \sum_{n=0}^{\infty} (n+2) a_n x^{n+1} + (x-2) \sum_{n=0}^{\infty} a_n x^{n+2}$$

$$\Rightarrow \begin{cases} a_0 = 0 \\ 3a_1 = 0 \end{cases}$$

$$z = -1: y = \frac{a_0}{x} + a_1 + a_2 x + \dots$$

$$\Rightarrow y'(x) = -\frac{a_0}{x^2} + a_2 + 2a_3 x + \dots$$

$$y''(x) = \frac{2a_0}{x^3} + 2a_3 + 6a_4 x + \dots$$

$$\Rightarrow \begin{cases} \frac{2a_0}{x^3} + 2a_3 x^2 + 6a_4 x^3 + \dots + a_0 - a_2 x - 2a_3 x^2 \\ + a_1 + a_2 x + a_3 x^2 + \dots - 2 \frac{a_0}{x} - 2a_1 - 2a_2 x = 0 \end{cases}$$

$$\begin{cases} a_0 + a_0 - 2a_1 = 0 & a_1 = a_0 \\ a_1 - 2a_2 = 0, & a_2 = \frac{1}{2} a_1 \\ 2a_3 - a_2 + 0 - 2a_3 = 0 & | a_3 = \frac{1}{2} a_2 \end{cases}$$

$$2) a_0 + \frac{1}{2} a_0 x + \frac{1}{4} a_0 x^2 + \dots$$

Крит. задача

Решить с помощью

$$\frac{dy_i}{dt} = f_i(t, y_1, \dots, y_n), \quad i = \overline{1, n}$$

$$y_i(t_0) = y_{i0}, \quad i = \overline{1, n}$$

$\varphi_i(t), i = \overline{1, n}$ - устр. по денудов, если

$$\forall \varepsilon > 0, \exists \delta(\varepsilon) > 0, |y_{i0} - \varphi_i(t-t_0)| < \delta(\varepsilon)$$

$$\Rightarrow \forall t > T, |y_i(t) - \varphi_i(t)| < \varepsilon$$

$t \rightarrow \infty, \Rightarrow 0$. 2) асимпт. устр

§8.1a)

$$\begin{cases} (t-1)\dot{x} = x & x(t) = 0 \\ x(2) = 0 \end{cases}$$

$$\begin{cases} (t-1)\dot{x} = x & \frac{dx}{x} = \frac{dt}{t-1} \\ x(2) = 0 \end{cases}$$

$$\Rightarrow \ln|x(t)| = \frac{1}{t-1} \ln|t-1|$$

$$\bar{x}(t) < c \sqrt[t]{t-1}$$

$$\delta = c$$

$$\bar{x}(t) = \sqrt[t]{t-1}; \quad \sqrt[t]{t-1} < \varepsilon \Rightarrow$$

$$t = 1 + \left(\frac{\varepsilon}{\delta}\right)^t < T$$

$$\forall \varepsilon > 0, \delta = \frac{\varepsilon}{\sqrt[t]{t-1}} \quad | \bar{x}(t=2) - 0 | = \delta$$

$$\Rightarrow |\bar{x}(t) - 0| < \varepsilon, \quad t > T.$$

$$\bar{x}(t) = \sqrt[t]{t-1} \quad \exists \delta > 0, \varepsilon > 1$$

$$\Rightarrow |\bar{x}(2) - 0| = 1 < \delta \Rightarrow \sqrt[t]{t-1} > 1$$

$$t = 1 + \left(\frac{\varepsilon}{\delta}\right)^t$$

$$\Rightarrow t > 1 + \left(\frac{1}{\delta^3}\right): |\bar{x}(t) - 0| > \varepsilon = 1.$$

$$\delta) \begin{cases} \dot{x} = 4x - t^2 x & x(t) = 0 \\ x(0) = 0 \end{cases}$$

$$\begin{cases} \dot{x} = 4x - t^2 x & \frac{dx}{x} = (4 - t^2) x \\ \bar{x}(0) = c \end{cases}$$

$$\Rightarrow \ln|x| = 4t - \frac{t^3}{3}$$

$$\bar{x} = c e^{4t - \frac{t^3}{3}}$$

$$f(t) = c e^{4t - \frac{t^3}{3}}$$

$$f(t) \rightarrow 0$$

$$\frac{f(t)}{t} \rightarrow 0$$

$$\begin{array}{c} \xrightarrow{t} \\ -2 \quad 2 \quad t-\frac{1}{2} \end{array}$$

$$\Rightarrow |f(t)| \leq e^{t-\frac{1}{2}} = e^{\frac{t}{2}}$$

$$|f(t)| \leq |t| e^{\frac{t}{2}} < \varepsilon$$

$$|t| < \frac{\varepsilon}{e^{\frac{t}{2}}}, \quad |t| < \frac{\varepsilon}{e^{\frac{t}{2}}}$$

for $\varepsilon > 0$, $\exists \delta > 0$ such that $\forall t$:

$$\begin{cases} |f(t)| = \frac{1}{2} e^{\frac{t}{2}} \\ |f(t)| = 0 \end{cases}$$

$$|f(t)| = 0$$

$$\left(\frac{1}{2} e^{\frac{t}{2}} \right) = \frac{1}{2} e^{\frac{t}{2}} \Rightarrow e^{\frac{t}{2}} = 2t^2$$

$$\frac{1}{2} e^{\frac{t}{2}} = 2t^2 \Rightarrow e^{\frac{t}{2}} = 4t^2$$

$$e^{\frac{t}{2}} > 1 \Rightarrow t > \frac{1}{2} \ln 2 = \ln 2 > 0$$

$$|f(t)| < \delta$$

$$\frac{1}{2} e^{\frac{t}{2}} < |f(t)| < e^{\frac{t}{2}} < \varepsilon$$

$$\frac{1}{2} e^{\frac{t}{2}} = \frac{1}{2} e^{\frac{t}{2}}$$

$$\frac{1}{2} e^{\frac{t}{2}} = 0 \text{ for } t = 0 \text{ or } \infty$$

$$\frac{1}{2} e^{\frac{t}{2}} = 0$$

$$\frac{1}{2} e^{\frac{t}{2}} = 0 \Rightarrow \frac{1}{2} e^{\frac{t}{2}} = 0$$

$$\frac{1}{2} e^{\frac{t}{2}} = -t + c$$

$$y = c e^{\frac{1}{2} t} - c e^{\frac{1}{2} t}$$

$$y = a e^{\frac{1}{2} t} + b$$

$$\Rightarrow \begin{cases} a = 1, b = 0 \\ a = 0, b = 1 \end{cases}$$

$$y(t) = t + c e^{\frac{1}{2} t}, c = 0$$

$$y(t) = t$$

$$|y(t) - y(t)| = |c e^{\frac{1}{2} t}|$$

$$|d| < \delta; \delta e^{\frac{t}{2}} < \varepsilon, e^{\frac{t}{2}} < \frac{\varepsilon}{\delta}$$

$$\Rightarrow t = \ln \frac{\varepsilon}{\delta}$$

$$\frac{1}{2} e^{\frac{t}{2}}$$

$$y(t) = c_1 \omega_1^2 t - c_2 e^t$$

$$y(t) = c_1 t^2 e^{-t} + 2c_2$$

$$y(t) = 0 \text{ for } t = 0$$

$$\|f(t)\|_2 = \sqrt{f_1(t)^2 + f_2(t)^2}$$

$$t = 0: \sqrt{(c_1 - c_2)^2 + 4c_2^2} < \delta$$

where $t = 0$.

$$|f(t) - g(t)| = \sqrt{(c_1 \omega_1^2 t - c_2 e^t)^2 + (c_1 t^2 e^{-t} + 2c_2)^2}$$

$$\leq c_1^2 \omega_1^4 t^2 + c_2^2 e^{2t} \leq (c_1^2 \omega_1^4 + c_2^2) t^2$$

$$\leq c_1^2 \omega_1^4 + c_2^2$$

$$\frac{1}{2} e^{\frac{t}{2}} = \frac{1}{2} e^{\frac{t}{2}} \Rightarrow e^{\frac{t}{2}} = 2t^2$$

$$\frac{1}{2} e^{\frac{t}{2}} = 2t^2 \Rightarrow e^{\frac{t}{2}} = 4t^2$$

$$\frac{1}{2} e^{\frac{t}{2}} = 0 \text{ for } t = 0$$

$$0 \leq \sqrt{(c_1 \omega_1^2 t - c_2 e^t)^2 + (c_1 t^2 e^{-t} + 2c_2)^2} < \varepsilon$$

$$\Rightarrow |c_1| < \frac{\varepsilon}{\omega_1^2 t}, |c_2| < \frac{\varepsilon}{2e^t}$$

$$\Rightarrow \delta \sqrt{\left(\frac{1}{2} e^{\frac{t}{2}} + 2\right)^2} < \varepsilon \Rightarrow \delta < \frac{\varepsilon}{\sqrt{\left(\frac{1}{2} e^{\frac{t}{2}} + 2\right)^2}}$$

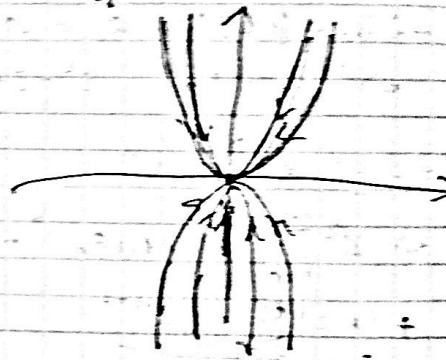
$$\frac{1}{2} e^{\frac{t}{2}}$$

$$\begin{cases} \dot{x} = -x \\ \dot{y} = -y \end{cases} \Rightarrow \frac{dx}{dt} = -x \Rightarrow x = -t + c$$

$$\Rightarrow y = e^{-t} c_1$$

$$\Rightarrow y = e^{-t} c_2$$

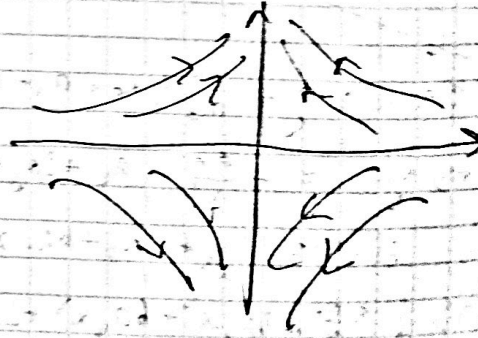
$$\Rightarrow y = x \frac{c_2}{c_1}$$



$$\frac{1}{2} e^{\frac{t}{2}}$$

$$\begin{cases} \dot{x} = x \\ \dot{y} = y \end{cases} \Rightarrow x = e^t c_1, y = e^t c_2$$

$$\Rightarrow y = \frac{c_2}{c_1} x$$



$$\frac{1}{2} e^{\frac{t}{2}}$$

$$y^{(4)} + 2y^{(3)} + 4y^{(2)} + 2y^{(1)} + 2y = 0$$

$$y_1(t) = e^{-t}, y_2(t) = e^{-t}, y_3(t) = e^{-t}, y_4(t) = e^{-t}$$

$$a_0 t^4 + a_1 t^3 + a_2 t^2 + a_3 t + a_4 = 0$$

Реш. $a_1 > 0$ | Если $n=2 \Rightarrow a_1 > 0$
(по 2-му кр. п. н. д.)
yes. $a_1 > 0$ yes.

Решение Кривой:

$$\begin{pmatrix} 2 & 1 & 0 & 0 \\ 3 & 4 & 2 & 1 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} -c a_1 \\ -c a_2 \\ -c a_3 \\ -c a_4 \end{pmatrix}$$

Анализ Рун. чисел, если все
мажор. $\Rightarrow$ yes. 1-й маж. 2-й маж.

$$\Delta_1 > 0, \Delta_2 > 0, \Delta_3 > 0.$$

899

$$y'' + ay' + by = 0$$

$$\begin{pmatrix} a & 1 & 0 \\ 2 & b & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$\Delta_1 = \Delta_2 > 0$$

$$\Delta_2 = ab - 2 > 0, b > \frac{2}{a} \text{ - аккумуля.}$$

$$\Delta_3 = 2b > 0$$

yes.

901

$$\begin{cases} \dot{x} = e^{x+2y} - \cos 3x \\ \dot{y} = \sqrt{4+8x} - 2e^y \end{cases}$$

Един Рели. 0
Крив.

$$\begin{aligned} e^{x+2y} &\sim 1+x+2y+ \\ \cos 3x &\sim 1- \\ \sqrt{4+8x} &\sim 2+2x+ \\ e^y &\sim 1+y+ \end{aligned}$$

$$\begin{cases} \dot{u} = 1+4+20-1=4+20 \\ \dot{v} = 2+2x-2-20=2x-20 \end{cases}$$

Решение
уравнения
Кривой
 $\Rightarrow$ аккумуля.

$$\begin{pmatrix} 1 & 2 \\ 2 & -2 \end{pmatrix}, \begin{pmatrix} 1-\lambda & 2 \\ 2 & -2-\lambda \end{pmatrix} = 0 \Rightarrow (\lambda-1)(\lambda+2)-4=0$$

$$\lambda^2 + \lambda - 6 = 0, \lambda_1 = 2, \lambda_2 = -3$$

-6 < 0 $\Rightarrow$ аккумуля.

899

$$\begin{cases} \dot{x} = 2xy - x + y \\ \dot{y} = 5x^2 + y^3 + 2x - y \end{cases} \begin{cases} \dot{u} = -u + v \\ \dot{v} = 2u - 3v \end{cases}$$

$$\begin{pmatrix} -1 & 1 \\ 2 & -3 \end{pmatrix} \Rightarrow (\lambda+1)(\lambda-3)-2=0$$

$$\lambda^2 - 2\lambda - 5 = 0$$

$\Rightarrow$ аккумуля.

2/2

881 (1,2)

$$\begin{cases} \dot{x} = t - x \\ x(0) = 1 \end{cases}$$

$$\dot{x} + x = t, x = ce^{-t} + t - 1$$

$$x(0) = c - 1 = 1 \Rightarrow c = 2$$

$$x(t) = 2e^{-t} + t - 1$$

$$\dot{x} = t - x \Rightarrow \dot{x} = t - (2e^{-t} + t - 1) = 1 - 2e^{-t}$$

$$x(0) = 1$$

$$ce^0 + 1 = x_0, c = 4x_0$$

$$\Rightarrow |(1+x_0)e^{-t} + t - 2e^{-t} - t + 1| < \frac{1}{8}$$

$$|x_0 e^{-t} - e^{-t}| < \frac{1}{8}, |x_0 + 1| e^{-t} < \frac{1}{8}$$

$$\Rightarrow \delta e^{-t} < \frac{1}{8}, -t < \ln \frac{1}{8\delta}$$

$$t > \ln \frac{8}{\delta} \text{ yes.}$$

881 (2)

$$\begin{cases} 2tx = x - x^3 \\ x(1) = 0 \end{cases}$$

$$\frac{dx}{x(x^2-1)} = -\frac{1}{2t} dt$$

$$\frac{x}{x^2-1} + \frac{1}{x} \Rightarrow \int \frac{dx}{x^2-1} - \ln|x| =$$

$$\frac{1}{2} \ln \left| \frac{x^2-1}{x^2} \right| = -\frac{1}{2} \ln|t| + C$$

$$\frac{(x^2-1)^2}{x^2} = C t^{-2} \Rightarrow \frac{x^2-1}{x^2} = C \frac{1}{t} \Rightarrow$$

$$\Rightarrow 1 - \frac{1}{x^2} = C \frac{1}{t} \Rightarrow 1 - C \frac{1}{t} = \frac{1}{x^2}, x^2 = \frac{1}{1-C \frac{1}{t}}$$

$$x = \pm \frac{1}{1-C \frac{1}{t}}, x(1) = \pm \frac{1}{1-C} = 0 \perp$$

888

$$\begin{cases} \dot{x} = x \\ \dot{y} = 2y \end{cases} \Rightarrow \frac{dy}{dx} = 2 \Rightarrow y = e^{2x} \cdot C_2$$

$$\Rightarrow y = \frac{1}{x^2} \cdot \frac{C_2}{C_1}$$



891

$$\begin{cases} x = \frac{c_1 - c_2 t}{1+t^2} \\ y = (c_1 e^t + c_2) e^{-t} \end{cases}$$

ln. S. T.
808
2H

$$|x|_{t=0} = |c_1| < 5 \Rightarrow |c_1| < 5$$

$$\Rightarrow \sqrt{c_1^2 + c_2^2} < 5 \Rightarrow |c_1| < 5, |c_2| < 5$$

$$\Rightarrow \sqrt{\left(\frac{1}{1+t^2}\right)^2 (c_1 - c_2 t)^2 + e^{-2t} (c_1 e^t + c_2)^2} < 5$$

$$\left| \frac{c_1^2}{(1+t^2)^2} - 2c_1 c_2 t \frac{1}{1+t^2} + \frac{c_2^2 t^2}{(1+t^2)^2} \right| \leq$$

$$\leq (|c_1| + |c_2|)^2 \Rightarrow 2T.g.$$

890

$$\begin{cases} \dot{x} = e^{x+2y} - \cos 3x \\ \dot{y} = 3x^2 - x + 3y \end{cases}$$

$$e^{x+2y} = 1 + e^{2y} x + 2y + \dots$$

$$\Rightarrow \begin{cases} \dot{u} = u + 2v - 1 \\ \dot{v} = u + 3v \end{cases} \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$$

$$\Rightarrow \lambda^2 - 4\lambda + 5 = 0 \Rightarrow \text{no real roots.}$$

902

$$\begin{cases} \dot{x} = \ln(1+y) + e^{3x} \\ \dot{y} = 2y - 1 + \frac{1}{1+6x} \end{cases}$$

$$\ln(1+x) = x - \frac{x^2}{2} + \dots$$

$$\Rightarrow \begin{cases} \dot{x} = 4y + e^{3x} - 1 - \frac{1}{2}(4y + e^{3x})^2 \\ \dot{y} = 2y - 1 + \frac{1}{1+6x} \end{cases}$$

$$\begin{cases} \dot{u} = 4v - 3u \\ \dot{v} = 2v - 2u \end{cases} \begin{bmatrix} -3 & 4 \\ -2 & 2 \end{bmatrix}$$

$$(-1-\lambda)(2-\lambda) + 8 = 0$$

$$= \lambda^2 + \lambda + 2 = 0$$

902

890

Кл. работа

881(2)

$$\begin{cases} 2tx = x - x^2, x(1) = 0 \\ x(t) = 0 \end{cases}$$

$$x(1) = 0$$

$$u = \frac{1}{x}, \dot{u} = -\frac{\dot{x}}{x^2}; -\dot{u} = \frac{u}{t} - \frac{1}{t}$$

$$\Rightarrow u = \frac{1}{ct} + 1 = \frac{1+ct}{ct}$$

$$x = \pm \sqrt{\frac{ct}{1+ct}}, x(1) = 0$$

$$c = \frac{1}{1-\delta^2} \quad x(t) \rightarrow 1 \quad \exists \varepsilon = \frac{1}{2};$$

$$\exists T: \forall t > T, |x(t) - 1| < \frac{1}{2}$$

903

$$\begin{cases} \dot{x} = y^2 - 2xy - 2y - x \\ \dot{y} = 2x + 2y^2 + e^{2t} - 2y \end{cases}$$

$$\begin{cases} x(t) = -t^2 \\ y(t) = t \end{cases} \text{ перемещаемся на } y=0$$

$$\begin{cases} x = u - t^2 \\ y = v + t \end{cases} \Rightarrow \begin{cases} \dot{u} - 2t = y^2 - 2xy - 2y - x \\ \dot{v} + 1 = 2(u - t^2) + 2(v + t)^2 + e^{2t} - 2(v + t) \end{cases}$$

$$\dot{u} - 2t = (v+t)^2 - 2t(v+t) - 2(v+t) - u + t^2$$

$$\dot{u} + 1 = 2(u - t^2) + 2(v + t)^2 + e^{2t} - 2(v + t)$$

$$\dot{u} - 2t = v^2 - 2v - 2t - u$$

$$\dot{v} + 1 = 2u + 2t^2 + 4t - 2v + \dots$$

$$\begin{bmatrix} -1 & -2 \\ 2 & -2 \end{bmatrix} \Rightarrow (-1-\lambda)(-2-\lambda) + 4 = 0$$

$$= \lambda^2 + 3\lambda + 2 = 0 \Rightarrow \lambda = -1, -2$$

for $\lambda = -1, -2 \Rightarrow$ see Red. 10

597

$$\begin{cases} \dot{x} = ax - 2y + x^2 \\ \dot{y} = x + y + xy \end{cases} \quad \begin{bmatrix} a & -2 \\ 1 & 1 \end{bmatrix} \Rightarrow (a-1)x - 1y + 2 = \lambda^2 + (-1-a)\lambda + a + 2$$

$$\begin{cases} -1-a > 0, a < -1 \\ a+2 > 0, a > -2 \end{cases} \quad (-1, 2)$$

598

$$\begin{cases} \dot{x} = y + \sin x \\ \dot{y} = ax + by \end{cases} \Rightarrow \lambda^2 + (b+1)\lambda - a + b$$

599

$$\begin{cases} \dot{x} = \ln(1+ax) - 1 \\ \dot{y} = bx + cy \end{cases} \Rightarrow \begin{cases} \dot{u} = 1 + \frac{a}{c}u - (1+v) \\ \dot{v} = bu + v \end{cases}$$

$$\begin{bmatrix} \frac{a}{c} - 1 & 1 \\ b & 1 \end{bmatrix} \Rightarrow \lambda^2 - \lambda(\frac{a}{c} + 1) + b + \frac{a}{c}$$

$$\begin{cases} 1 + \frac{a}{c} > 0 \\ \frac{a}{c} + 1 < 0 \end{cases} \Rightarrow \begin{cases} b > -\frac{a}{c} \\ a < -c \end{cases}$$

5921

$$\begin{cases} \dot{x} = \ln(1+y+\sin x) \\ \dot{y} = 2 + \frac{1}{3}\sin x - 8 \end{cases}$$

numerically

$$\begin{cases} 1+y+\sin x = 1 \\ 2 + \frac{1}{3}\sin x - 8 = 0 \end{cases} \Rightarrow (\pi, 0), n \in \mathbb{Z}$$

$$\begin{cases} x = u + \pi n \\ y = v \end{cases} \Rightarrow \begin{cases} \dot{u} = \ln(1+v + (-1)^n \sin u) \\ \dot{v} = 2 + \frac{1}{3}(-1)^n \sin u - 8 \end{cases}$$

$$\ln(1+v + (-1)^n \sin u) = \ln(1+v) + (-1)^n \frac{\sin u}{1+v} + \dots$$

$$\begin{cases} \dot{u} = v + (-1)^n u \\ \dot{v} = \frac{(-1)^n}{4} u \end{cases} \Rightarrow \begin{bmatrix} (-1)^n & 1 \\ \frac{(-1)^n}{4} & 0 \end{bmatrix} \Rightarrow$$

$$\Rightarrow (1-1)^n - 1(-1) - \frac{(-1)^n}{4}, n=2k \Rightarrow \text{negligible}.$$

5914

$$\begin{cases} \dot{x} = \ln(x + 2\sin^2 \frac{t}{2}) - \frac{t}{2} \\ \dot{y} = (4-x^2)\cos t - 2x \cdot \sin^2 t - \cos^3 t \end{cases}$$

$$\begin{cases} x = \cos t, & \dot{x} = -\sin t \\ y = 2\sin t, & \dot{y} = 2\cos t \end{cases}$$

$$\begin{cases} \dot{u} - \sin t = \ln(u + \cos t + 2\sin^2 \frac{t}{2}) - \frac{1}{2}(v + 2\sin t) \\ \dot{v} + 2\cos t = (4 - (u + \cos t)^2 + 4\cos t - 2(u + \cos t)\sin^2 t - \cos^3 t) \end{cases}$$

$$\begin{cases} \dot{u} = \ln(u+1) - \frac{v}{2} \\ \dot{v} = 2\cos t - u^2 \cos t - 2u \cos^2 t - \cos^3 t - 2u \sin^2 t - 2\cos t \sin^2 t - \cos^3 t - 2\cos t \end{cases}$$

$$\begin{cases} \dot{u} = u^2 - u - \frac{v}{2} \\ \dot{v} = -2u - u^2 \cos t \end{cases} \Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} \\ -2 & 0 \end{bmatrix}$$

$$\Rightarrow \lambda^2 - \lambda - 1 \Rightarrow \text{negligible}$$

2/p

5908

$$\begin{cases} \dot{x} = ax + y + x^2 \\ \dot{y} = x + ay + y^2 \end{cases} \quad \begin{bmatrix} a & 1 \\ 1 & a \end{bmatrix} \Rightarrow (a-\lambda)^2 - 1 = a\lambda^2 - 2a\lambda + a - 1$$

$$\begin{cases} -2a > 0, a < 0 \\ a^2 - 1 > 0, a < -1 \end{cases}$$

311

$$\begin{cases} \dot{x} = 2e^{-x} - \sqrt{4+ay} \\ \dot{y} = \ln(1+x+ay) \end{cases} \Rightarrow \begin{cases} \dot{x} = 2(1-x) - \frac{2}{3}(1+ay)^{\frac{1}{2}} \\ \dot{y} = \ln(1+x+ay) \end{cases}$$

$$\begin{cases} \dot{u} = x - 2x - \frac{1}{4}ay \\ \dot{v} = x + ay \end{cases} \Rightarrow \begin{bmatrix} -2 & -\frac{1}{4}a \\ 1 & a \end{bmatrix}$$

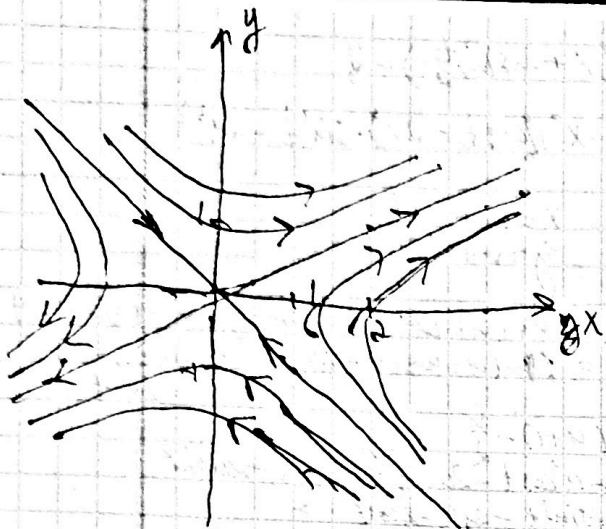
$$\Rightarrow (-2-1)(a-1) + \frac{1}{4}a = \lambda^2 + \lambda(2-a) - \frac{a^2}{4}$$

Imp. p. 153

5961

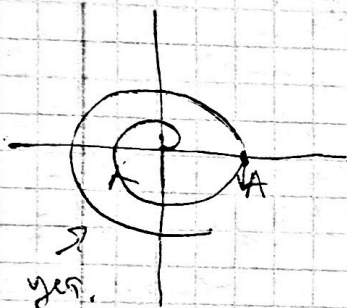
$$\begin{cases} \dot{x} = 3x + 4y \\ \dot{y} = 2x + y \end{cases} \Rightarrow \begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix} \Rightarrow \lambda^2 - 4\lambda - 5 = 0 \Rightarrow \lambda_1 = 5, \lambda_2 = -1$$

$$\lambda = -1: \begin{bmatrix} 4 & 4 \\ 2 & 2 \end{bmatrix} \Rightarrow e_1 = (1, -1), e_2 = (2, 1)$$



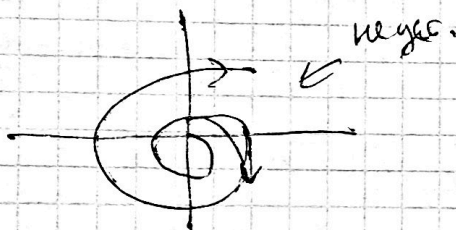
963

$$y' = \frac{y-2x}{y} \quad \begin{cases} \dot{x} = y \\ \dot{y} = y-2x \end{cases} \Rightarrow \lambda^2 - \lambda + 1$$



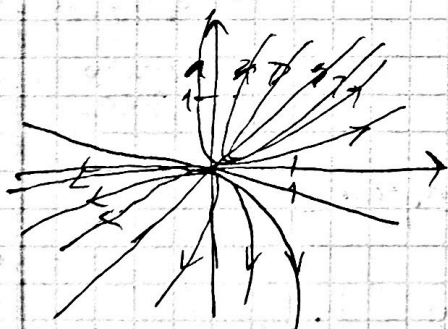
$\Rightarrow$ Periodic
Re $\lambda > 0$ - unstable
Re $\lambda < 0$ - stable
yes.

$$(\bar{x}, \bar{y}) = (0, -2)$$



964

$$y' = \frac{x+4y}{2x+y} \quad \begin{cases} \dot{x} = x+4y \\ \dot{y} = 2x+y \end{cases} \Rightarrow \lambda = -5, \lambda = +1$$



yes. stable.

Asymptotically stable.

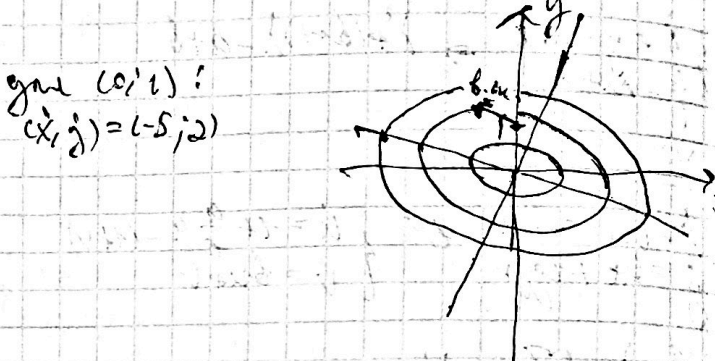
962

965

$$\begin{cases} \dot{x} = -2x + 5y \\ \dot{y} = 2x + 2y \end{cases} \quad \begin{vmatrix} -2-\lambda & 5 \\ 2 & 2-\lambda \end{vmatrix} =$$

$$\lambda^2 - 4 + 10 = \lambda^2 + 6 \quad \text{Red } > 0 \Rightarrow \text{unstable.}$$

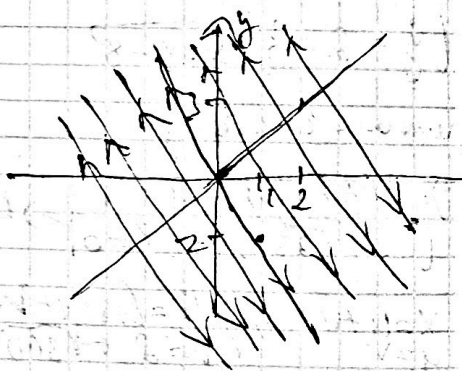
$$f(x,y) = x^2 + y^2, \quad \frac{df}{dt} = 2x\dot{x} + 2y\dot{y}$$



967

$$\begin{cases} \dot{x} = 3x - 2y \\ \dot{y} = 4y - 6x \end{cases} \quad \begin{vmatrix} 3-\lambda & -2 \\ -6 & 4-\lambda \end{vmatrix} \Rightarrow \lambda = 0, \lambda = 7$$

$$\lambda = 0: e_1(2,1), \lambda = 7: e_2(1,-2)$$



981

$$y' = \frac{4y^2 - x^2}{2xy - 4y - 8} \quad \begin{cases} \dot{x} = 2xy - 4y - 8 \\ \dot{y} = 4y^2 - x^2 \end{cases}$$

$$\rightarrow A = (4,2), B = (-2,-1)$$

$$\begin{cases} x = u + 4 \\ y = v + 2 \end{cases} \quad \begin{cases} \dot{u} = 2(u+4)(v+2) - 4(u+4) - 8 \\ \dot{v} = 4(v+2)^2 - (u+4)^2 \end{cases}$$

$$\begin{cases} \dot{u} = 2uv + 4v + 6 - 7v + 4u - 4v \\ \dot{v} = 4u^2 + 16v - 4u - 8u \end{cases}$$

$$\Rightarrow \begin{cases} \dot{u} = 4u + 4v \\ \dot{v} = 4u + 16v \end{cases} \quad \begin{bmatrix} 4 & 4 \\ -8 & 16 \end{bmatrix} \quad (4-\lambda)(16-\lambda) + 32 = 0$$

$$\Rightarrow \lambda^2 - 20\lambda + 96 = 0 \Rightarrow \frac{8}{4} = 4 \quad \lambda_{1,2} = 10 \pm 2 \sqrt{16}$$

$$\text{B: } \begin{cases} \dot{u} = 2(u-2)(v-1) - 4(v-1) - 8 = (v-1)(2u-4-4) - 8 \\ \dot{v} = 4(v-1)^2 - (u-2)^2 = (2v-2-u+2)(2v-2+u-2) \end{cases}$$

$$\begin{cases} \dot{u} = -2u - 8v \\ \dot{v} = -8v + 4u \end{cases} \Rightarrow \lambda = -5 \pm \sqrt{25}; \text{ - qo'k'ye } \text{Re } \lambda < 0 \Rightarrow \text{yur.}$$

$$\begin{cases} \dot{x} = x^2 - y \\ \dot{y} = \ln(1-x+x^2) - \ln 3 \end{cases}$$

$$\begin{cases} x^2 - y = 0 \\ \ln(1-x+x^2) = \ln 3 \end{cases} \Rightarrow y = x^2, \quad 1-x+x^2 = 3 \Rightarrow x^2 - x - 2 = 0 \Rightarrow x_{1,2} = \frac{1 \pm 3}{2} = \begin{cases} 2 \\ -1 \end{cases}$$

$$A(1, 1), B(2, 4) \quad \begin{cases} x = u+2 \\ y = v+4 \end{cases} \Rightarrow \begin{cases} \dot{u} = (u+2)^2 - (v+4) \\ \dot{v} = \ln\left(\frac{1+4+2+(u+2)^2}{3}\right) \end{cases}$$

$$\dot{v} = \ln\left(\frac{u^2 + 4u + 3}{3}\right) \Rightarrow \begin{cases} \dot{u} = 4u - v \\ \dot{v} = u \end{cases}$$

$$\Rightarrow \begin{bmatrix} 4-\lambda & -1 \\ 1 & 0-\lambda \end{bmatrix} \quad (4-\lambda)(-\lambda) + 1 = \lambda^2 - 4\lambda + 1$$

$$\lambda_{1,2} = \frac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3}$$

kerf. yur.

$$(-1, 1): \begin{cases} x = u-1 \\ y = v+1 \end{cases} \Rightarrow \begin{cases} \dot{u} = (u-1)^2 - (v+1) \\ \dot{v} = \ln\left(\frac{1-u+1+(u-1)^2}{3}\right) \end{cases}$$

$$\ln(\dots) = \ln\left(\frac{3+u^2-3u}{3}\right)$$

$$\Rightarrow \begin{cases} \dot{u} = -2u - v \\ \dot{v} = -4 \end{cases} \quad \begin{bmatrix} -2 & -1 \\ -1 & 0 \end{bmatrix} \Rightarrow (-2-\lambda)(-\lambda) + 1 = 0$$

$$2\lambda + \lambda^2 + 1 = 0 \Rightarrow \lambda = -1 \pm \sqrt{2}$$

$$\lambda = \begin{bmatrix} -1 + \sqrt{2} \\ -1 - \sqrt{2} \end{bmatrix}$$

kerf. yur.

§96

$$\begin{cases} \dot{x} = \ln(2-y^2) \\ \dot{y} = e^x - e^y \end{cases} \Rightarrow \begin{cases} 2-y^2 = 1, y = \pm 1 \\ e^x = e^y, x = y-1 \end{cases}$$

$$\begin{cases} x = u+1 \\ y = v+1 \end{cases} \Rightarrow \begin{cases} \dot{u} = \ln(1-v^2-2v) \\ \dot{v} = e^{u+1} - e^{v+1} \end{cases}$$

$$\begin{cases} \dot{u} = -2v \\ \dot{v} = u e - v e \end{cases} \quad \begin{vmatrix} -1 & -2 \\ e & -e-1 \end{vmatrix} = (e+1)e + 2e = 0$$

$$\Rightarrow \lambda^2 + e\lambda + 2e, \lambda_{1,2} = \frac{-e \pm \sqrt{e^2 - 8e^2}}{2} = \frac{-e \pm \sqrt{-7e^2}}{2}$$

$$(-1, -1): \begin{cases} \dot{u} = 2v \\ \dot{v} = u e - v e \end{cases} \quad \begin{vmatrix} -1 & 2 \\ e & -e-1 \end{vmatrix} = \lambda^2 + e\lambda + 2e = 0$$

$$\lambda_{1,2} = \frac{-e \pm \sqrt{e^2 - 8e^2}}{2}$$

kerf. yur.

§97

§95

$$\begin{cases} x - 2y = \dot{x} \\ 3x - 4y = \dot{y} \end{cases} \Rightarrow \begin{cases} \dot{x} = 3x - 4y \\ \dot{y} = x - 2y \end{cases}$$

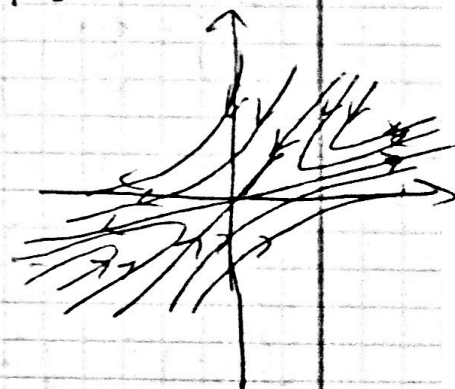
$$\begin{bmatrix} 3 & -4 \\ 1 & -2 \end{bmatrix} \rightarrow (3-\lambda)(-2-\lambda) + 4 = 0$$

$$= \lambda^2 - \lambda - 2 = (\lambda+1)(\lambda-2)$$

kerf. $\lambda > 0: 2 \Rightarrow \dots$

$$\lambda = 2: \begin{bmatrix} 1 & -4 \\ 1 & -2 \end{bmatrix} \Rightarrow e_1(4, 1)$$

$$\lambda = -1: \begin{bmatrix} 4 & -4 \\ 1 & -1 \end{bmatrix} \Rightarrow e_2(1, 1)$$



§970

$$\begin{cases} \dot{x} = 3x - 4y \\ \dot{y} = x - 2y \end{cases} \quad \begin{bmatrix} 3 & -4 \\ 1 & -2 \end{bmatrix}$$

$$(3-\lambda)(-2-\lambda) + 4 = \lambda^2 - \lambda - 2, \lambda_{1,2} = \frac{1 \pm 3}{2}$$

$$(\dot{x}, \dot{y}) = (3, 4)$$

kerf.



977

$$\begin{cases} \dot{x} = x + by \\ \dot{y} = -6x - 5y \end{cases} \begin{bmatrix} 1 & 3 \\ -6 & 5 \end{bmatrix} (1-\lambda)(5-\lambda) + 18 = 0$$

$\Rightarrow$ gauge neg.

976

$$\begin{cases} \dot{x} = bx + y \\ \dot{y} = y - x \end{cases} \begin{bmatrix} 3 & 1 \\ -1 & 1 \end{bmatrix} (3-\lambda)(1-\lambda) + 1 = 0$$

$\Rightarrow \lambda^2 - 4\lambda + 4 = 0, \Delta = 16 - 16 = 0$
 $(\lambda - 2)^2 = 0$
 $\lambda = 2$

254
282
284
756
758
760
762

$$\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \Rightarrow e_1(1, -1)$$

$\Rightarrow$ gauge neg.

$$(x, y) \begin{pmatrix} 1 & 2 \\ -2 & 6 \end{pmatrix}$$



983

$$y' = \frac{x^2 + y^2 - 2}{x - y} \begin{cases} \dot{x} = x - y \\ \dot{y} = x^2 + y^2 - 2 \end{cases}$$

$x^2 - 2y^2 = 2, y = \pm 1 \Rightarrow x = \pm 1$
 $(1, 1)$
 $(-1, -1)$

$x = u + 1, y = v + 1 \Rightarrow \begin{cases} \dot{u} = u - v \\ \dot{v} = 2u + 2v \end{cases}$

$$\begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix} \Rightarrow \lambda^2 - 3\lambda + 4 = 0$$

$\Rightarrow$ gauge neg.

$$(-1, -1): \begin{cases} \dot{u} = u - v \\ \dot{v} = -2u - 2v \end{cases}$$

$$(1-\lambda)(-2-\lambda) - 2 = \lambda^2 + 3\lambda - 4 = 0$$

$\Rightarrow$ gauge neg.

987

$$\begin{cases} \dot{x} = (2x - y)(x - 2) \\ \dot{y} = xy - 2 \end{cases} \Rightarrow \begin{cases} 4 = y^2, x = 1 \\ y = \pm 2, y = 1 \end{cases}$$

$\Rightarrow x = \pm 1$
 $\Rightarrow (1, 2)$
 $(-1, -2)$
 $(2, 1)$

$$x = u + 1, y = v + 1$$

$$\Rightarrow \begin{cases} \dot{u} = (2(u+1) - (v+1))(u+1 - 2) \\ \dot{v} = (u+1)(v+1) - 2 \end{cases} \Rightarrow \begin{cases} \dot{u} = -2u + v + 4u - 2 \\ \dot{v} = (u+1)(v+1) - 2 \end{cases}$$

$$\Rightarrow \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \Rightarrow$$

2/P
Cute virus -

751

Sp

§ 751

$$\begin{cases} y'' - y = 2x \\ y'(0) = 0, \\ y(1) = -1 \end{cases} \quad 1) \lambda^2 - 1 = 0, \lambda = \pm 1 \Rightarrow y_0 = C_1 e^x + C_2 e^{-x}$$

$$y_2 = ax + b \Rightarrow y_2'' = 0 \Rightarrow \frac{-ax - b}{2} = 2x \Rightarrow a = -2 \Rightarrow y_2 = -2x$$

$$\Rightarrow y = C_1 e^x + C_2 e^{-x} - 2x$$

$$\begin{cases} C_2 + C_1 = 0 \\ C_1 e + \frac{1}{e} C_2 = -2 = -1 \end{cases} \Rightarrow \begin{cases} C_1 = -C_2 \\ -C_2(e + \frac{1}{e}) = -1, C_2 = \frac{1}{e + \frac{1}{e}}, C_1 = \frac{1}{e + \frac{1}{e}} \end{cases}$$

$$\Rightarrow y = \frac{1}{e + \frac{1}{e}} (e^x - e^{-x}) - 2x \quad \text{Antwort}$$

§ 752

$$\begin{cases} y'' + y' = 1 \\ y'(0) = 0 \\ y(1) = 1 \end{cases} \quad 1) \lambda^2 + \lambda = 0, \lambda(\lambda + 1) = 0 \Rightarrow \begin{matrix} \lambda = 0 \\ \lambda = -1 \end{matrix} \Rightarrow y_0 = C_1 + C_2 e^{-x}$$

$$y_2 = ax + b \Rightarrow y_2' = a$$

$$\Rightarrow y = C_1 + C_2 e^{-x} + x, y' = -C_2 e^{-x} + 1$$

$$\begin{cases} -C_2 + 1 = 0 \\ C_1 + C_2 e^{-1} + 1 = 1, C_2 = 1, C_1 = -e^{-1} \end{cases} \Rightarrow y = -e^{-1} + e^{-x} + x \quad \text{Antwort}$$

§ 754

$$\begin{cases} y'' + y = 1 \\ y'(0) = 0 \\ y(\frac{\pi}{2}) = 0 \end{cases} \quad 1) \lambda^2 + 1 = 0, \lambda = \pm i \Rightarrow y_0 = C_1 \sin x + C_2 \cos x$$

$$y = C_1 \sin x + C_2 \cos x + 1 \quad \begin{cases} C_2 + 1 = 0 \\ C_1 + 1 = 0 \end{cases} \Rightarrow y = 1 - \sqrt{2} \sin(x + \frac{\pi}{4})$$

§ 756

$$\begin{cases} y'' + y = 2x - \pi \\ y(0) = 0 \\ y(\pi) = 0 \end{cases} \quad 1) \lambda^2 + 1 = 0, \Rightarrow y = C_1 \sin x + C_2 \cos x + x - \pi$$

$$y_2 = 2x - \pi$$

$$\begin{cases} C_2 - \pi = 0, C_2 = \pi \\ -C_2 + \pi - \pi = 0 \end{cases} \Rightarrow C_2 = \pi, C_1 = -\pi$$

$$y = C_1 \sin x + C_2 \cos x + x - \pi \quad \text{Antwort}$$

§ 758

$$\begin{cases} y'' - y = 1 \\ y(0) = 0 \\ |y(x)| < \epsilon \quad x \rightarrow \infty \end{cases} \quad y = C_1 e^x + C_2 e^{-x} + 1 \quad \begin{cases} C_1 + C_2 - 1 = 0 \Rightarrow C_2 = 1 \\ C_1 = 0 \end{cases}$$

$$y = e^{-x} - 1$$

§ 760

$$\begin{cases} x^2 y'' - 6y = 0 \\ y(0) = 2 \\ y'(0) = 0 \end{cases} \quad x = e^t \Rightarrow y' = \dot{y} e^{-t}, y'' = e^{-2t} (\ddot{y} - \dot{y}) \cdot e^{-t} = e^{-2t} (\ddot{y} - \dot{y})$$

$$\Rightarrow \ddot{y} - \dot{y} - 6y = 0 \quad \lambda^2 - \lambda - 6 = 0, \lambda_{1,2} = \frac{1 \pm 5}{2} = 3, -2$$

$$y = C_1 e^{3t} + C_2 e^{-2t}$$

$$y = C_1 x^3 + C_2 x^{-2} \quad \Rightarrow$$

$$\begin{cases} c_1 + c_2 = 2, c_1 = 2 \Rightarrow y = 2x^2 \\ c_2 = 0 \end{cases}$$

5762

$$\begin{cases} x^2 y'' + 5xy' + 3y = 0 \\ y(1) = 3 \\ y(x) = 0 \text{ for } x \rightarrow +\infty \end{cases}$$

$$\begin{aligned} \ddot{y} - \dot{y} + 5(y) + 3y &= 0, \ddot{y} + 4\dot{y} + 3y = 0 \\ \lambda^2 + 4\lambda + 3 &= 0 \Rightarrow \lambda = \frac{-4 \pm 2}{2} = -2 \pm 1 = -1, -3 \\ \Rightarrow y &= c_1 x^{-1} + c_2 x^{-3} \\ y' &= -c_1 x^{-2} - 3c_2 x^{-4} \end{aligned}$$

Df.

766

$$\begin{aligned} y'' + y' &= f(x) \\ y(0) &= 0 \\ y(1) &= 0 \end{aligned} \Rightarrow \begin{cases} c_1 + c_2 = 0 \\ -c_1 e^{-1} = 0 \end{cases} \Rightarrow \begin{cases} c_1 = 0 \\ c_2 = 0 \end{cases} \Rightarrow y_1 = 1 - e^{-x} \\ y_2 = 1 \end{aligned}$$

$$\Rightarrow \begin{cases} (1 - e^{-1})c_1 = c_2, c_2 = (e^3 - 1) \\ c_1 e^{-1} = 1, c_1 = e \end{cases} \Rightarrow G(x, s) = \begin{cases} \frac{1}{e} (1 - e^{-x}), & 0 \leq x \leq s \\ (e^3 - 1), & s \leq x \leq 1 \end{cases}$$

769

$$\begin{aligned} x^2 y'' + 2xy' &= f(x) \\ y(1) &= 0 \\ y(3) &= 0 \end{aligned} \quad x = e^t \Rightarrow y' = e^{-t} \dot{y} \Rightarrow y'' = e^{-2t} (\ddot{y} - \dot{y})$$

$$\Rightarrow \ddot{y} - \dot{y} + 2\dot{y} = 0, \ddot{y} + \dot{y} = 0 \Rightarrow y = c_1 e^{-t} + c_2$$

$$y = c_1 x^{-1} + c_2$$

$$\Rightarrow \begin{cases} c_1 + c_2 = 0 \\ -\frac{c_1}{x^2} = 0 \end{cases} \Rightarrow \begin{cases} c_1 = 0 \\ c_2 = 1 \end{cases} \Rightarrow G(x, s) = \begin{cases} c_1 (x^{-1} - 1), & 1 \leq x \leq s \\ c_2, & s \leq x \leq 3 \end{cases}$$

$$\Rightarrow \begin{cases} c_1 (\frac{1}{3} - 1) = c_2, c_2 = (1 - \frac{1}{3}) \\ -\frac{c_1}{3^2} = -\frac{1}{3^2}, c_1 = 1 \end{cases} \Rightarrow G(x, s) = \begin{cases} (1 - x^{-1}), & 1 \leq x \leq s \\ (1 - \frac{1}{3}), & s \leq x \leq 3 \end{cases}$$

771

$$\begin{aligned} y'' + y' &= f(x) \\ y(0) &= 0 \\ y(+\infty) &= 0 \end{aligned} \quad y = c_1 e^{-x} + c_2$$

$$\begin{cases} c_1 = 0 \\ c_2 = 0 \end{cases} \Rightarrow y_1 = 1, y_2 = e^{-x} \Rightarrow G(x, s) = \begin{cases} c_1, & 0 \leq x \leq s \\ c_2 e^{-x}, & s \leq x < \infty \end{cases}$$

$$\begin{cases} c_1 = c_2 e^{-s} \\ 0 + 1 = -c_2 e^{-s}, c_2 = -e^s \end{cases} \Rightarrow G(x, s) = \begin{cases} -1, & 0 \leq x \leq s \\ -e^{s-x}, & s \leq x < \infty \end{cases}$$

775

11

$$\begin{cases} y'' + 4y' + 3y = f(x) \\ y(0) = 0 \\ y(x) = O(e^{-x}), x \rightarrow +\infty \end{cases}$$

$$1) \lambda^2 + 4\lambda + 3 = 0$$

$$\lambda_{1,2} = \frac{-4 \pm \sqrt{4}}{2} = -2 \pm 1$$

$$y = c_1 e^{-x} + c_2 e^{-3x}$$

$$\begin{cases} c_1 + c_2 = 0 \Rightarrow y_1 = e^{-x} - e^{-3x} \\ c_1 = 0 \Rightarrow y_2 = e^{-3x} \end{cases}$$

$$G(x, s) = \begin{cases} c_1 (e^{-x})(1 - e^{-3s}), & 0 \leq x \leq s \\ c_2 e^{-3x}, & s \leq x < +\infty \end{cases}$$

$$\begin{cases} c_1(1 - e^{-3s}) = c_2 e^{-3s} \\ c_1(-e^{-s} + 1)e^{-3s} + 1 = -3e^{-3s} c_2, & c_1 = e^{-2s}(c_2 + c_1), & c_2 + c_1 = e^{2s} c_1 \\ -e^{-s} c_1 + 1 = e^{-3s}(-3)(c_2 + c_1), & +e^{-s} c_1 + 1 = +3e^{-s} c_1, & 2e^{-s} c_1 = -1 \end{cases}$$

$$c_1 = -\frac{1}{2} e^s$$

$$c_2 = c_1(e^3 - 1)$$

$$c_2 = -\frac{1}{2} e^s (e^3 - 1)$$

$$\Rightarrow G(x, s) = \begin{cases} -\frac{1}{2} e^s (e^{-x} - e^{-3x}), & 0 \leq x \leq s \\ -\frac{1}{2} e^s (e^3 - 1) e^{-3x}, & s \leq x < +\infty \end{cases}$$

751

$$\begin{cases} y'' - y = 2x \\ y(0) = 0 \\ y(1) = -1 \end{cases}$$

$$\lambda^2 - 1 = 0, \lambda = \pm 1 \Rightarrow y = c_1 e^x + c_2 e^{-x}$$

$$\begin{cases} c_1 + c_2 = 0 \\ c_1 e + c_2 e^{-1} = -1, & c_1 = -c_2 \\ c_2 = -e \end{cases} \Rightarrow y = -e \cdot e^x$$

$$G(x, s) = \begin{cases} c_1 (e^x - e^{-x}), & 0 \leq x \leq s \\ c_2 (-e \cdot e^{-x}), & s \leq x \leq 1 \end{cases}$$

$$\begin{cases} c_1 (e^s - e^{-s}) = c_2 (-e) \cdot e^{-s} \\ c_1 (e^s + e^{-s}) + 1 = c_2 e e^{-s} \end{cases}$$

$$c_1 (2e^s) + 1 = 0, c_1 = -\frac{1}{2} e^{-s}$$

$$c_2 = e^s \frac{1}{2} c_1 (e^3 - e^{-3})$$

$$c_2 = -\frac{1}{2} e^s (e^3 - e^{-3})$$

$$\Rightarrow G(x, s) = \begin{cases} -\frac{1}{2} e^{-s} (e^x - e^{-x}), & 0 \leq x \leq s \\ +\frac{1}{2} e^s (e^3 - e^{-3}) (-e \cdot e^{-x}), & s \leq x \leq 1 \end{cases}$$

$$y_{\text{part}} = \int_0^1 (G(x, s) 2s) ds = \int_0^1 -\frac{1}{2} e^{-s} (e^x - e^{-x}) s^2 ds + \int_0^1 e^{-x} s^2 (e^3 - e^{-3}) ds$$

$$\int_0^1 e^{-s} ds = -e^{-s} \Big|_0^1 = e^{-s} (1 - 0) = 1 - \frac{1}{e}$$

$$\int_0^1 e^{-s} ds = (1 - \frac{1}{e}) - e^{-s} (s - 1) \Big|_0^1 = 1 - \frac{1}{e} + 1 \Rightarrow y_{\text{part}} = \left\{ \dots \right\}$$

753

$$\begin{cases} y'' - y' = 0 \\ y(0) = 1 \\ y(1) - y(1) = 2 \end{cases}$$

$$y(x) = u(x) + v(x) \Rightarrow \begin{cases} u'' - u' = 0 \\ u(0) = 0 \\ u(1) - u(1) = 0 \end{cases}$$

$$\Rightarrow \lambda^2 - \lambda = 0, \lambda = 0, 1$$

$$u = c_1 + c_2 e^x$$

$$\begin{cases} u_1 = 1 - e^x \\ u_2 = e^x \end{cases}$$

$$\Rightarrow \begin{cases} c_1 = -1 \\ 2a + b - a + b - c = 2 \\ a = 2 + c + 2b \\ -a = 1 \\ \Rightarrow 2(x) = x - 1 \end{cases}$$

$$\Rightarrow G(x, s) = \begin{cases} c_1 (1 - e^x), & 0 \leq x \leq s \\ c_2 e^x, & s \leq x \leq 1 \end{cases}$$

$$\begin{cases} c_1(1-e^s) = c_2 e^s \\ c_1 = c_2 e^s + c_2 e^s \\ c_2 e^s + 1 = c_2 e^s \Rightarrow 1 = c_2 e^s + c_2 e^s \end{cases}$$

$$\Rightarrow y(x) = \int_0^1 G(x,s) ds + x^2 - 1$$

$$\begin{cases} c_1 = 1 \\ 1-e^s = c_2 e^s \Rightarrow c_2 = e^{-s} - 1 \end{cases}$$

$$G(x,s) = \begin{cases} 1-e^x, & 0 \leq x \leq s \\ (e^s-1)e^x, & s \leq x \leq 1 \end{cases}$$

751

$$\begin{cases} y'' - y = 2x \\ y(0) = 0 \\ y(1) = -1 \end{cases}$$

$$y = u - x \Rightarrow \begin{cases} u'' - u = x \\ u(0) = 0 \\ u(1) = 0 \end{cases}$$

$$\lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1$$

$$y = c_1 e^x + c_2 e^{-x}$$

$$\begin{cases} c_1 + c_2 = 0 \Rightarrow y_1 = e^x - e^{-x} \\ e c_1 + c_2 \frac{1}{e} = 0 \Rightarrow c_2 = -e^2 c_1 \Rightarrow y_2 = e^x - e^2 e^{-x} \end{cases}$$

$$\begin{cases} c_1 e^s (1-e^{-2s}) = c_2 e^s (1-e^{-2s}) \\ c_1 e^s (1+e^{-2s}) + 1 = c_2 e^s (1+e^{-2s}) \end{cases} \Rightarrow \begin{cases} c_1 = c_2 \\ 2c_1 e^{-2s} + 1 = 2c_2 e^{-2s} \end{cases}$$

$$G(x,s) = \begin{cases} c_1(e^x - e^{-x}) & 0 \leq x \leq s \\ c_2(e^x - e^{2-x}) & s \leq x \leq 1 \end{cases}$$

$$\Rightarrow G(x,s) = \begin{cases} \frac{1}{2} \frac{e^2}{(1-e^2)} (e^x - e^{-x}), & 0 \leq x \leq s \\ \frac{1}{2} \frac{1}{1-e^2} (e^x - e^{2-x}), & s \leq x \leq 1 \end{cases}$$

$$\Rightarrow 2c_1 e^{-2s} (1-e^2) = e^{2-2s} \Rightarrow c_1 = \frac{1}{2} \frac{e^2}{(1-e^2)}$$

$$c_2 = \frac{1}{2} \left(\frac{e^2 + 1}{1-e^2} \right)$$

$$\Rightarrow y(x) = \int_0^1 G(x,s) ds + x$$

752

$$\begin{cases} y'' - y' - 2y = 0 \\ y'(0) = 2 \\ y(x \rightarrow \infty) = 0 \end{cases} \quad y = u - 2e^{-x} \Rightarrow u'' - 2e^{-x} - u' + 2e^{-x} - 2u + 4e^{-x} = 0, \quad u'(0) = 0, \quad u(x \rightarrow \infty) = 0$$

$$\lambda^2 + \lambda - 2 = 0 \Rightarrow (\lambda+2)(\lambda-1) = 0 \Rightarrow u = c_1 e^{-x} + c_2 e^{2x}$$

$$1) u_1 = e^{-x} + e^{2x}$$

$$u_2 = e^{-x}$$

$$\Rightarrow G(x,s) = \begin{cases} c_1(e^{-x} + e^{2x}) & 0 \leq x \leq s \\ c_2 e^{-x} & s \leq x \rightarrow \infty \end{cases}$$

$$\begin{cases} c_1(e^{-s} + e^{2s}) = c_2 e^{-s} \\ c_1(e^{-s} + e^{2s}) + 1 = -c_2 e^{-s} \end{cases}$$

$$\Rightarrow \begin{cases} c_1 e^{-\frac{1}{2}s} \\ c_2 = -\frac{1}{2} (e^s + e^{-s}) \end{cases} \Rightarrow$$

$$G(x, s) = \begin{cases} -\frac{1}{2}e^{-2s}(e^{-x} + e^x) & 0 \leq x \leq 3 \\ -\frac{1}{2}(e^3 - e^{2s})e^{-x} & 3 \leq x \leq +\infty \end{cases}$$

$$\Rightarrow y(x) = \int_0^{+\infty} G \cdot 0.6s \cdot 2e^{-2x} ds = -2e^{-2x}$$

Dif

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